The Origins of Firm Heterogeneity: A Production Network Approach*

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June 2019

Abstract

This paper explores firm size heterogeneity in an interconnected production network. Using all buyer-supplier relationships in Belgium, the paper shows that firms with more customers have higher sales but lower sales per customer and market shares among those customers. A decomposition of firm sales reveals that downstream factors, especially the number of customers, explain the large majority of firm size heterogeneity. These facts motivate a model of heterogeneous firms and network formation where firms search for, and sell to, downstream buyers and buy inputs from upstream suppliers. Firms vary in their productivity and relationship capability. Higher productivity results in greater market shares among customers and more matches. Higher relationship capability results in more customers and greater sales. Estimates of model parameters show that relationship capability and productivity are strongly negatively correlated. The results suggest that models with a single dimension of firm heterogeneity, such as quality or productivity, cannot match key features of the production network.

JEL: D24, F10, F12, F16, L11, L25

Keywords: Firm size heterogeneity, relationship capability, productivity, production network, network formation, matching costs.

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*This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreements No 715147 and No 724880). The views expressed here are those of the authors and do not necessarily reflect those of the National Bank of Belgium. We thank discussants Julian di Giovanni, Isabelle Mejean, Jasmine Xiao and Thomas Åstebro as well as various seminar participants and two anonymous referees at the National Bank of Belgium for helpful comments and valuable discussion.

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1 Introduction

Even within narrowly defined industries, there is massive dispersion in firm outcomes such as sales, employment and labor productivity. In Belgium, a firm at the 90th percentile of the size distribution has turnover more than 32 times greater than a firm at the 10th percentile in the same industry.[1] Understanding the origins of firm size heterogeneity is a fundamental question in economics and has important micro and macroeconomic implications. At the micro level, bigger firms perform systematically better along many dimensions, including survival, innovation, and participation in international trade (e.g., Bernard et al., 2012). At the macro level, the skewness and granularity of the firm size distribution affect aggregate productivity, the welfare gains from trade, and the impact of idiosyncratic and systemic shocks (e.g., Pavcnik, 2002, Gabaix, 2011, di Giovanni et al., 2014, Melitz and Redding, 2015 and Gaubert and Itskhoki, 2016).

This paper examines the firm size distribution in a complete production network with firm heterogeneity and buyer-supplier connections.[2] The basic premise of the analysis is intuitive: firms can be large because they have inherently attractive capabilities such as productivity or quality, because they have low marginal costs due to more or better suppliers, and/or because they have higher sales due to more or bigger buyers. There may be higher-order effects in the production network as well, because the customers of one's customers may ultimately also matter for the firm's economic outcomes.

While research has made progress in identifying underlying firm-specific supply- and demand-side factors driving firm size (e.g., Hottman et al., 2016), much less is known about the role of buyer-seller linkages in production networks. In particular, the focus on the supply side has been on heterogeneity in either firm productivity (e.g., Jovanovic, 1982, Hopenhayn, 1992, Melitz, 2003, Luttmer, 2007) or organizational capital (e.g., Prescott and Visscher, 1980, Luttmer, 2011), whereas work on the demand side has centered on final customer preferences (e.g., Fitzgerald et al., 2016) or firm-specific demand stocks (e.g., Foster et al., 2016, Eckel et al., 2015). To the extent that the literature has considered firm-to-firm trade, it has typically remained anchored in one-sided heterogeneity by assuming that firms source inputs from anonymous upstream suppliers or sell to anonymous downstream buyers, without accounting for the heterogeneity of all trade partners in the production network.

Our contribution is threefold. First, we document new stylized facts about a complete

[1] Averaged across all NACE 4-digit Belgian industries in 2014. Hottman et al. (2016) report even larger sales dispersion across firms using barcode data on US retail sales, and Autor et al. (2017) find that 52.6 percent of sales in the average US industry is accounted for by the 20 largest firms, representing less than half a percent of the total number of firms.

[2] Throughout the paper, firm size refers to sales or turnover, and the terms are used interchangeably.

production network using data on the universe of firm-to-firm domestic transactions in Belgium, and present the first extensive analysis of how upstream, downstream and final demand heterogeneity translate into firm size heterogeneity. Second, we provide a theoretical framework of an endogenous production network with firm heterogeneity in both productivity/quality and relationship capability. Third, we estimate the parameters of the model using simulated method of moments to explore the relative importance of the two dimensions of firm heterogeneity and their interaction across firms. We leverage unique data on firm-to-firm sales between virtually all firms in the domestic production network in Belgium.

We report three stylized facts from the production network data that motivate the subsequent analysis and model. First, the distributions of total sales, buyer-supplier connections and buyer-supplier bilateral sales exhibit high dispersion and skewness. The enormous dispersion found in sales across firms is also found in the production network in terms of links and the value of pairwise sales. Second, firms with more customers have higher sales but lower average sales per customer and lower market shares (shares of input purchases) among their customers. Finally, there is negative degree assortativity between buyers and suppliers, i.e. sellers with more customers match with customers who have fewer suppliers on average.

Taken together, these facts both confirm intuition and challenge existing models of firm heterogeneity. The large variation in sales across firms within an industry is intuitively related to variation in the number of customers: large firms have more customers. However, firms with more customers have lower average sales per customer, connect with less well-connected customers, and account for a smaller share of those customers' input purchases. Models that emphasize heterogeneity in productivity across firms cannot explain all three facts simultaneously. In particular, such models imply that firms with more customers should also sell more to each of their customers: they should have higher, not lower market shares.

A key advantage of the production network data is that sales from firm i to j can be decomposed into seller-, buyer- and match specific components (fixed effects).[3] High dispersion in seller effects means that firms vary in how much they sell to their customers, controlling for demand by those customers, i.e. firms differ in their average market share across customers. Conversely, high dispersion in buyer effects means that some firms match with large customers while others do not, leading to larger sales even as the average market share remains the same. Given estimates of these fixed effects, the total sales of a firm can be decomposed into three distinct factors: (i) an upstream component that captures the firm's ability to obtain large market shares across its customers, (ii) a downstream component that captures the firm's ability to attract many and/or large customers, and (iii) a final demand component that captures the firm's ability to sell relatively more outside the

[3] This is similar to the analysis of employer-employee data (Abowd et al., 1999).

domestic network, to final consumers at home or to foreign customers.

The results are striking: 76 percent of the variation in firm sales within narrowly defined (4-digit NACE) industries is associated with the downstream component, while the upstream component contributes only 24 percent. The variation in firm size is largely unrelated to the final demand component. These findings imply that trade in intermediate goods and the number of firm-to-firm connections are essential to understanding firm performance and, consequently, aggregate outcomes.

Motivated by these stylized facts and decomposition results, we develop a quantitative general equilibrium model of firm-to-firm trade. In the model, firms use a constant-elasticity-of-substitution production technology that combines labor and inputs from upstream suppliers. Firms sell their output to final consumers and to domestic producers. Firms differ in two dimensions – productivity and relationship capability – defined respectively as production efficiency and (the inverse of) the fixed cost of matching with a customer. The two dimensions are potentially correlated. Suppliers match with customers if the gross profits of the match exceed the supplier-specific fixed matching cost. Marginal costs, employment, prices, and sales are endogenous outcomes because they depend on the outcomes of all other firms in the economy. A link between two firms increases the total sales of both the seller and the buyer; for the seller this occurs mechanically because it gains a customer, while for the buyer this arises because a larger supplier base lowers the marginal cost of production.

The solution of the network model involves three nested fixed points. A backward fixed point determines the price of a firm as a function of its marginal cost, which in turn depends on the prices of its suppliers. A forward fixed point pins down the sales of a firm as a function of demand by its customers, which in turn depends on their sales to their customers. A link function fixed point relates the likelihood of a link to the profit from the match, which is itself a function of the network structure.

We estimate the parameters of the model using simulated method of moments. These parameters comprise the mean and variance of relationship capability, the variance of productivity, and the correlation between productivity and relationship capability across firms. The corresponding moments we target in the data are the mean and variance of the log number of customers, the variance of sales per customer, and the coefficient from a regression of log sales per customer on log number of customers.

The results reveal a strong negative correlation between the two firm characteristics and high dispersion in relationship capability across firms. Firms with higher productivity have lower relationship capability. This negative relationship is crucial for matching the stylized facts that firms with more customers have lower average sales per customer and lower market share in those customers. The model does well at matching untargeted moments on the downstream side, such as the variances of total sales, sales in the network and negative degree assortativity. In addition, it does well at matching moments on the upstream side of the production network including the variances of the number of suppliers and total input purchases, the positive relationship between purchases per supplier and the number of suppliers, and upstream degree assortativity. Importantly, both dimensions of firm heterogeneity are necessary to match the data: shutting down one at a time results in poor model fit, including the inability to replicate the negative relationship between the number of customers and average sales per customer.

Moreover, while canonical models of firm heterogeneity (e.g., Hsieh and Klenow, 2009) are unable to generate dispersion in labor productivity within an industry in the absence of capital and/or output distortions, our model produces this as an equilibrium outcome because more productive and/or high relationship capability firms have greater value added per marketing worker (i.e., workers allocated to relationship building).

This paper contributes to several strands of literature. Most directly, the paper adds to the large literature on the extent, causes and consequences of firm size heterogeneity. The vast dispersion in firm size has long been documented, with recent emphasis on the skewness and granularity of firms at the top end of the size distribution (e.g., Gibrat, 1931, Syverson, 2011). This interest is motivated by the superior growth and profit performance of bigger firms at the micro level, as well as by the implications of firm heterogeneity and superstar firms for aggregate productivity, growth, international trade, and adjustment to various shocks (e.g., Gabaix, 2011, Bernard et al., 2012, Freund and Pierola, 2015, Gaubert and Itskhoki, 2016, Oberfield, 2018).

Traditionally, this literature has analyzed own-firm characteristics on the supply side as the driver of firm size heterogeneity. The evidence indicates an important role for firms' production efficiency, management ability, and capacity for quality products (e.g., Jovanovic, 1982, Hopenhayn, 1992, Melitz, 2003, Sutton, 2007, Bender et al., 2018, Bloom et al., 2017). Recent work has built on this by also considering the role of either upstream suppliers or downstream demand heterogeneity, but not both. Results suggest that access to inputs from domestic and foreign suppliers matters for firms' marginal costs and product quality, and thereby performance (e.g., Goldberg et al., 2010, Bøler et al., 2015, Manova et al., 2015, Fieler et al., 2018, Antràs et al., 2017, Boehm and Oberfield, 2018 and Bernard et al., 2019a), while final consumer preferences affect sales on the demand side (e.g., Foster et al., 2016, Fitzgerald et al., 2016).

By contrast, we provide a comprehensive treatment of both own firm characteristics and production network features, on both the upstream and the downstream sides. The paper is related to Hottman et al. (2016) who also find that demand-side factors such as variation in firm appeal and product scope rather than prices (marginal costs) drive firm size dispersion. However, as these authors do not observe the production network, they cannot distinguish between the impact of serving more customers, attracting better customers, and selling large amounts to (potentially few) customers. Since they have no information on the supplier margin, they also cannot separate own from network supply factors. On the other hand, while rich in network features, our data do not provide information on prices or products, and thus do not allow for a comparable decomposition into firm appeal and product scope.

The paper also adds to a growing literature on buyer-supplier production networks (see Bernard and Moxnes, 2018 for a recent survey). Bernard et al. (2019a) study the impact of domestic supplier connections on firms' marginal costs and performance in Japan, whereas Bernard et al. (2018a), Eaton et al. (2016), and Eaton et al. (2018) explore the matching of exporters and importers using data on firm-to-firm trade transactions for Norway, US-Colombia, and France, respectively. Using the Belgian production network data, Magerman et al. (2016) analyze the contribution of the network structure of production to aggregate fluctuations, while Tintelnot et al. (2017) examine the effect of trade on the domestic production network. In recent work, Baqaee and Farhi (forthcoming), Baqaee and Farhi (2018) and Lim (2018) consider how microeconomic shocks shape macroeconomic outcomes in networked environments. Our work departs from this literature by focusing on the dispersion of firm outcomes and their relationship to upstream and downstream features of the network.

The rest of the paper is organized as follows. Section 2 introduces the data and presents stylized facts about the Belgian production network. Section 3 decomposes firm sales into upstream, downstream and final demand components. Section 4 develops a theoretical framework with heterogeneous firms and endogenous matching in a production network. Section 5 estimates the parameters of the model and quantitatively assesses the two dimensions of firm heterogeneity. The last section concludes.

2 Data and Stylized Facts

2.1 Data

The empirical analysis draws on three comprehensive micro-level data sources, administered at the National Bank of Belgium (NBB). These include (i) information on the universe of domestic firm-to-firm relationships across all economic activities from the NBB B2B Transactions Dataset, (ii) firm characteristics from the annual accounts at the Central Balance Sheet Office, and (iii) the sector of main economic activity from the Crossroads Bank of Enterprises. Each firm has a unique legal identification number, allowing for unambiguous matching across the datasets. We use the 2014 cross-section for the main analysis. A detailed description of the construction and cleaning of the datasets is provided in Appendix A.

The primary source is the NBB B2B Transactions Dataset, which documents both the extensive and intensive margins of supplier-buyer relationships in Belgium.[4] In particular, an observation in this dataset refers to the sales value in euro, m_{ij}, of firm i selling to firm j within Belgium (net of the VAT amount due). The reported value is the sum of invoices from i to j in a given calendar year. Coverage is quasi universal, as all yearly relationships of at least 250 euros must be reported, and pecuniary sanctions by the tax authorities on late and erroneous reporting ensure high data quality. Each observation m_{ij} is directed, as firm i can be selling to j, but not necessarily the other way around.

Data on total sales (turnover), total input purchases, employment and labor costs come from firm annual accounts administered by the Central Balance Sheet Office (CBSO) at the NBB. Fiscal years have been annualized to calendar years to match the unit of observation in the NBB B2B data. Very small firms do not have to report turnover or input expenditures in their annual accounts. For these, we use information from an auxiliary data source, the VAT declarations.[5] We keep firms with at least one full-time equivalent employee resulting in a core sample of 94,334 firms. The main economic activity of each enterprise is available at the NACE 4-digit level (harmonized over time to the NACE Rev. 2 (2008) version) from the Crossroads Bank of Enterprises.

We define firms' sales to final demand as the difference between their total turnover and the sum of all their B2B sales to other enterprises in the domestic network. Final demand thus contains sales to final consumers at home and exports.[6] Firms' purchases from outside the observed production network (including imports) is the difference between their total input costs and the sum of all their B2B purchases.

2.2 Stylized Facts

This section documents three stylized facts about firm size and firm linkages in the Belgian production network.[7] These facts provide evidence that buyer-supplier relationships are key to understanding firm size dispersion, and motivate the subsequent theoretical and empirical

[4] See Dhyne et al. (2015) for details on the construction of this dataset.

[5] The periodic VAT declarations are the main source of information on VAT due for the tax authorities. They report the total amount sold and bought inside the VAT system for each firm, including imports and exports.

[6] Since we do not observe the identity of foreign buyers, we treat exports to both final consumers and downstream firms abroad as final demand from the perspective of the exporting firm.

[7] A subset of these stylized facts echo patterns established for the extensive margin of firm-to-firm linkages in the domestic production network in Japan (Bernard et al., 2019a) and for both the extensive and the intensive margins of firm-to-firm export transactions in Norway (Bernard et al., 2018a) and Belgium (Dhyne et al., 2015).

analyses. We present cross-sectional evidence for 2014, but the patterns are similar for other years.

Fact 1. *The distributions of firms' total sales, buyer-supplier connections, and buyer-supplier bilateral sales exhibit high dispersion and skewness.*

Firm size varies dramatically in Belgium, as in other countries. Table 1 provides summary statistics for firm sales, both overall and within six broad sectors (primary and extraction, manufacturing, utilities, construction, market services, and non-market services).[8] Across the 94,334 firms in our sample, average sales are 7.5 million euros, with a standard deviation of 156 million euros. Similar patterns hold within each broad sector category.

The cross-sectional distribution is extremely skewed. Within a 4-digit industry and averaged across industries, firms at the 90th percentile generate 32 times more salesthan firms at the 10th percentile, while the top 10 percent of firms account for 84 percent of total sales. Within narrowly defined industries, large firms are up to four orders of magnitude bigger than their industry mean, as shown in Figure 1.

Turning to firm-to-firm connections in the domestic production network, the number of downstream customers per seller (out-degree) and the number of upstream suppliers per buyer (in-degree) are very skewed. Table 2 summarizes the overall distribution of buyer and supplier connections. The average number of customers is 123, with a standard deviation of 801. The average number of suppliers is 79, with a standard deviation of 109. Firm-to-firm links in the network are also highly concentrated among a few very connected participants: The median number of customers and suppliers is 26 and 53 respectively, while the top 1 percent of firms transact with more than 1,200 buyers and 450 suppliers.

The intensive margin of firm-to-firm bilateral sales is also very dispersed and skewed, with the vast share of economic activity concentrated in a small number of buyer-supplier transactions. The mean transaction amounts to 30,594 euros with a standard deviation of 3.1 million euros. At the same time, the median is 1,417 euros, and the top 10% of relationships account for 92% of all domestic firm-to-firm sales by value.

Fact 2. *Firms with more customers have higher sales but lower sales per customer.*

A sharp pattern in the data is that bigger firms interact with more buyers in the production network. Figure 2a shows the binned scatterplot of firm sales to other producers in the network (y-axis) against the number of customers (x-axis), on a log-log scale. Both variables have been demeaned by their 4-digit industry average. The elasticity of sales with respect to the number of customers is 0.77. Across firms the number of customers rises faster than total

[8] See Table 12 in Appendix A for the classification of industry groups at the 2-digit NACE level.

Table 1: Firm Sales, million €

Sector	NACE	N	Mean	St Dev	10th	50th	90th	95th	99th
Primary & Extraction	01-09	2,838	12.8	449.3	0.2	0.8	5.1	9.9	57.6
Manufacturing	10-33	16,905	15.2	259.3	0.3	1.2	14.7	36.4	223.8
Utilities	35-39	852	37.2	441.5	0.4	1.9	26.8	68.5	495.5
Construction	41-43	19,008	2.3	13.8	0.2	0.6	3.7	7.1	27.6
Market Services	45-82	53,582	6.3	86.7	0.2	1.0	7.4	15.7	73.9
Non-Market Services	84-99	1,149	2.4	10.8	0.2	0.5	2.9	7.1	35.7
All		94,334	7.5	155.6	0.2	0.9	7.4	16.4	89.7

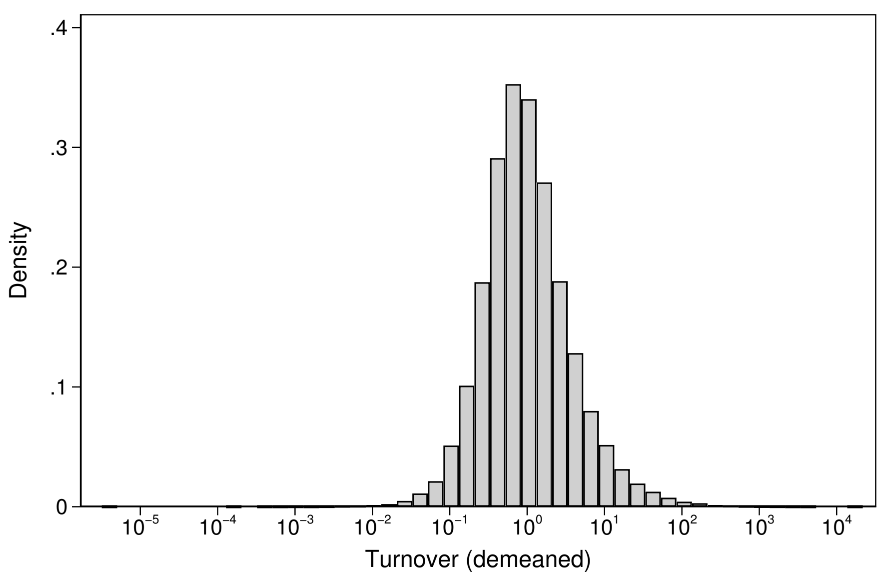
Note: 10th, 50th, etc. refers to values at the 10th, 50th, etc. percentile of the distribution.

Table 2: Degree Distributions

	N	Mean	St Dev	10th	50th	90th	95th	99th
# of buyers	94,334	123.4	800.8	3	26	245	429	1,297
# of suppliers	94,334	78.9	108.9	20	53	152	219	466

Note: 10th, 25th, etc. refers to values at the 10th, 25th, etc. percentile of the distribution.

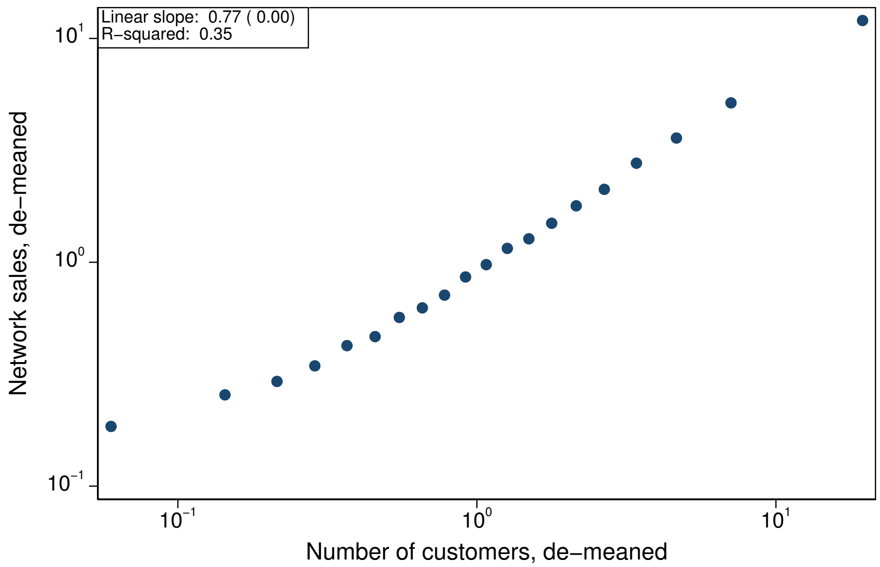
Figure 1: Firm Sales Distribution



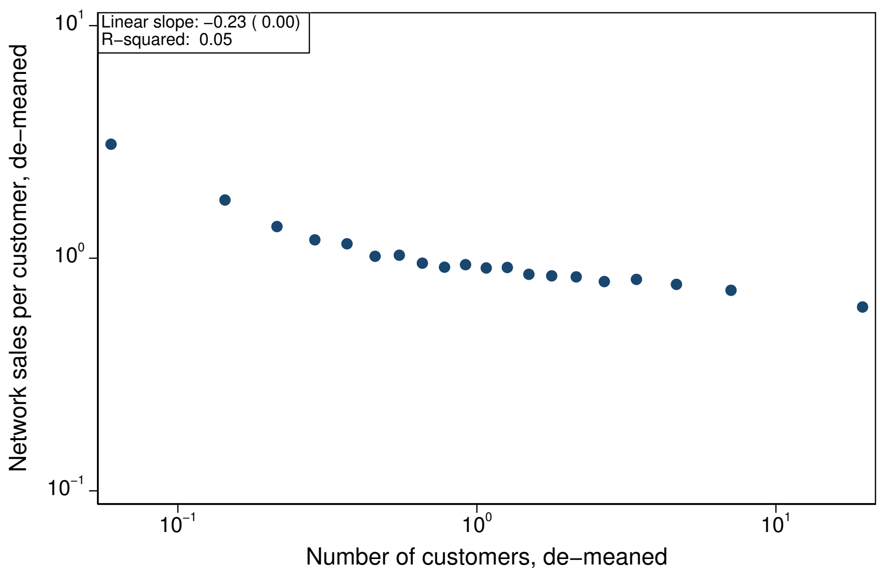
Note: The figure shows the density of within-industry firm-level sales on a log scale. Log sales are demeaned by NACE 4-digit industry averages.

Figure 2: Total Network Sales, Average Sales and Number of Customers

(a) Total Sales



(b) Average Sales



Note: The binned scatterplots group firms into 20 equal-sized bins by log number of customers, and compute the mean of the variables on the x- and y-axes in each bin. Network sales refer to sales to customers in the domestic production network. All variables are demeaned by NACE 4-digit industry averages. Implied elasticities and R-squared from OLS regressions with NACE 4-digit industry fixed effects are reported in the corner of each graph.

sales. This implies that the sales per customer is decreasing in the number of customers, as illustrated in Figure 2b.[9] Figure 3a documents that the decline in sales per customer takes place for both big and small customers. For each firm, we calculate the $10th$, $50th$ and $90th$ percentile of sales across its customers, and plot these percentiles against the firm's number of customers. The slope coefficients range between -.25 and -.29.

One may wonder if the decline in average sales per customer is driven by selection: if sellers with many customers tend to match with smaller buyers, they would also tend to have lower average and median bilateral sales. To address this, we leverage the production network data and calculate a firm's average market share among its customers, the geometric mean of m_{ij}/M_j, where m_{ij} is sales from i to j and M_j is total purchases by firm j. If selection were the main mechanism, this average market share would be increasing in, or unrelated to, the number of customers. Figure 4b shows that this is not the case; firms' average market share also declines with their number of customers, with an elasticity of -0.51, falling even faster than average sales.

Taken together, these empirical regularities present a puzzle: big firms match with many buyers, but they are unable to gain a large market share among those buyers. By contrast, in canonical models (e.g., Arkolakis, 2010, Bernard et al., 2018a, Eaton et al., 2018, Lim, 2018), highly productive firms would both attract many customers and have a high market share among those customers. The empirical evidence therefore calls for a model with an additional element of firm heterogeneity where firm size is not only determined by productivity, but also by a second firm attribute, which enables firms to match with more buyers.

Fact 3. *There is negative degree assortativity among sellers and buyers.*

An important property of networks is the extent to which a well-connected node is linked to other well-connected nodes, so-called degree assortativity. The production network is characterized by negative degree assortativity. In other words, better connected firms match to less well-connected firms on average.[10] Figure 4 shows a binned scatterplot of the average number of suppliers to firm i's customers on the y-axis against the number of i's customers, on a log-log scale. The fitted regression line has slope -0.05, such that doubling the number of customers is associated with a 5 percent decline in the average customer's number of suppliers. We also find a robust, and more negative, relationship between a firm's number of suppliers and the average supplier's number of customers (see Table 9).

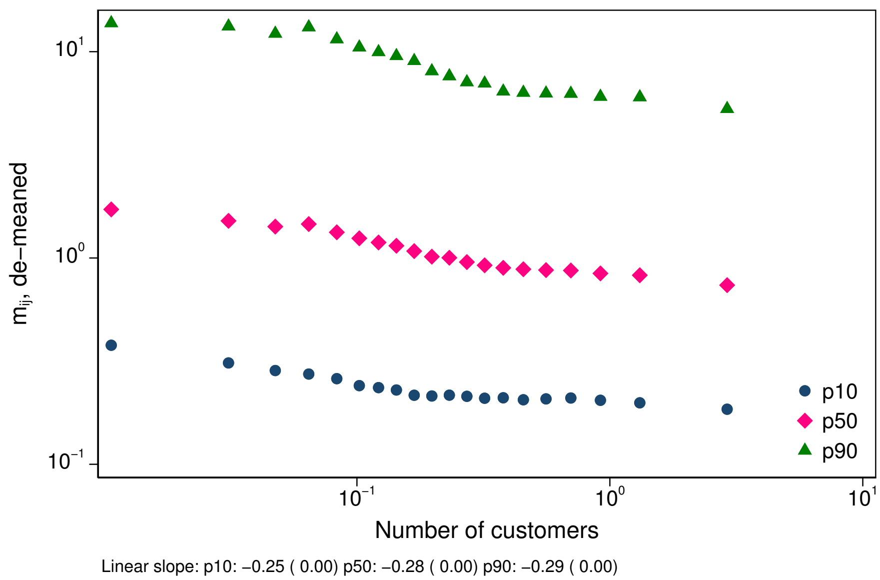
Negative degree assortativity motivates our choice of a parsimonious matching model, in which firm connections form whenever the gross profits of a match exceed a relationship

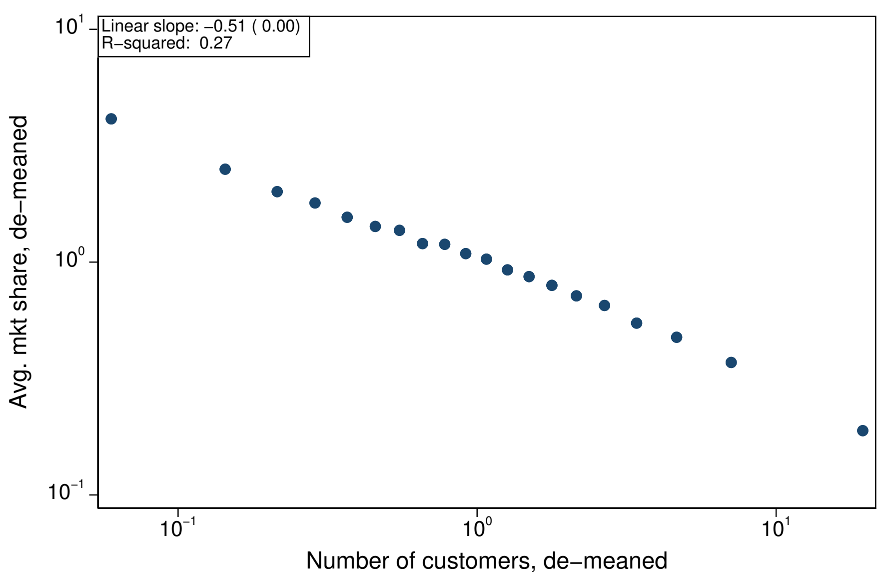
[9] Results are similar using total sales (instead of network sales), as shown in Appendix C.

[10] Negative degree assortativity has been documented in earlier research on production networks, e.g. Bernard et al. (2019a), Bernard et al. (2018a) and Lim (2018).

Figure 3: Sales Distribution, Market Share and Number of Customers

(a) Sales Distribution

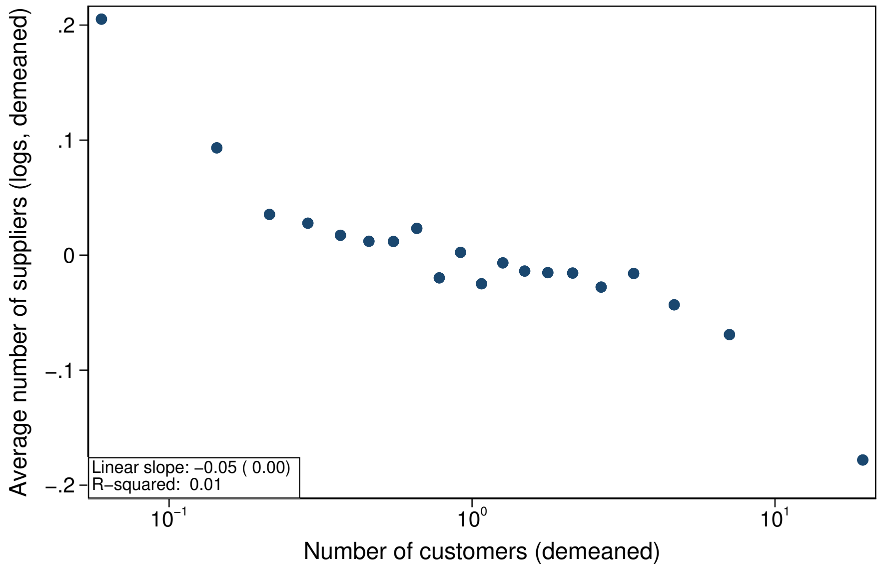




(b) Market Share

Note: The binned scatterplots group firms into 20 equal-sized bins by log number of customers, and compute the mean of the variables on the x- and y-axes in each bin. All variables are demeaned by NACE 4-digit industry averages. p90/p50/p10 refer to the $90th/50th/10th$ percentile of firm-to-firm sales m_{ij} across buyers j within firm i. Average market share refers to the geometric mean of the market share m_{ij}/M_j of firm i in the total input purchases of its buyers j. Implied elasticities and R-squared from OLS regressions with NACE 4-digit industry fixed effects are reported in the corner of each graph.

Figure 4: Degree Assortativity



Note: The binned scatterplot groups firms into 20 equal-sized bins by log number of customers, and computes the mean of the variables on the x- and y-axes in each bin. Average number of suppliers refers to the geometric mean of the number of suppliers serving the customers of firm i. All variables are demeaned by NACE 4-digit industry averages. Implied elasticities and R-squared from OLS regressions with NACE 4-digit industry fixed effects are reported in the corner of the graph.

fixed cost. In this class of models, the marginal (and average) customer of more capable firms is less capable, generating a pattern of negative degree assortativity. Appendix C.7 reports facts from the upstream side (e.g., the relationship between the number of suppliers and purchases). Upstream facts are also discussed in more detail in the analysis of model fit for untargeted moments in Section 5.2.

3 An Exact Decomposition

In this section, we develop an exact variance decomposition of firm sales into upstream, downstream, and final demand margins. The stylized facts from Section 2 and the results from the variance decomposition jointly inform the theoretical framework presented in Section 4. The variance decomposition will also be used to assess the model fit based on the structural estimation in Section 5.

The downstream component reflects characteristics of a firm's customers (i.e., their number and size), while the upstream component captures firm characteristics that remain constant across customers (i.e., average sales to customers, controlling for their size). Final demand includes factors unrelated to the domestic production network, such as sales to final consumers or foreign customers. This approach exploits the granularity of firm-to-firm transactions to inform the micro-foundations of firm size in a way that would be impossible without production network data.

3.1 Methodology

We start by estimating buyer, seller and buyer-seller match effects using data on sales between firms in the production network. The specification is a two-way fixed effects regression for (log) firm-to-firm sales:

$$\ln m_{ij} = G + \ln \psi_i + \ln \theta_j + \ln \omega_{ij}, \tag{1}$$

where $\ln m_{ij}$ is log sales from i to j and G is the grand mean of $\ln m_{ij}$ across all ij pairs. In this OLS regression, the seller effect $\ln \psi_i$ is identified by the magnitude of sales by i to all its customers j, controlling for total purchases by j. The seller effect is therefore the average market share of i among her customers. Intuitively, attractive sellers account for a large share of input expenditures across all their customers and receive a high $\ln \psi_i$. Analogously, the buyer effect $\ln \theta_j$ is identified by the magnitude of purchases by j from all its suppliers i, controlling for total sales of i. Intuitively, attractive buyers purchase a disproportionate share of suppliers' sales and receive a high $\ln \theta_j$. A positive residual $\ln \omega_{ij}$ reflects match-specific characteristics that induce a given firm pair to trade more with each other, even if they are not fundamentally attractive trade partners.

To interpret the variation in ψ_i and θ_j, consider the case where the variation in $\ln m_{ij}$ is only due to ψ_i. Sellers i and i' then differ because i sells more to every customer (while buyers j and j' purchase the same amount from i). Consider next the opposite case where the variance in $\ln m_{ij}$ is only due to θ_j. Sellers i and i' now differ because i happens to match with bigger customers than i' (while sales to a common customer j are identical). In the first case firm heterogeneity is driven by differences in sales ability, while in the second case it is driven by differences in matching ability.

To obtain unbiased OLS estimates, the assignment of suppliers to customers must be exogenous with respect to ω_{ij}, so-called conditional exogenous mobility (Abowd et al., 1999). This identification assumption, as well as tests for exogenous mobility and functional form relevance, are discussed in Appendix C. Overall, we find support for the log linear model and the conditional exogenous mobility assumption.

In order to estimate the two-way fixed effects model, firms must have multiple connections. Specifically, identifying a seller fixed effect requires a firm to have at least two customers, and identifying a buyer fixed effect requires a firm to have at least two suppliers. Therefore, single-customer and single-supplier links are dropped in the estimation procedure. Furthermore, dropping customer A might result in supplier B having only one customer left. Supplier B would then also be removed from the sample. This iterative process continues until a connected network component remains (i.e. a within-projection matrix of full rank), in which each seller has at least two customers and each buyer has at least two suppliers. This component is known as a mobility group in the labor literature on firm-employee matches.

The production network can be represented by a directed graph, with an edge denoting a sales relationship from seller i to buyer j. Moreover, firms can be simultaneously buyers and sellers. Production networks are therefore richer and more complex than typical bipartite networks that have been studied extensively, such as the labor market for firms and workers. In particular, each firm has both a seller and a buyer fixed effect which can be identified using cross-sectional production network data as long as the firm has enough upstream and downstream partners. This has two advantages. First, it attenuates the incidental parameter problem as the number of suppliers per customer and the number of customers per supplier is relatively large (see Section 2) . Second, this setting does not require the typical assumption that the fixed effects are constant over time.

Given estimates of $\Psi = \{\psi_i, \theta_j, \omega_{ij}\}$, firm sales can be exactly decomposed into upstream, downstream, and final demand factors. Total sales of firm i are by construction $S_i = \sum_{j \in \mathcal{C}_i} m_{ij} + \mathcal{F}_i$, where $\mathcal{C}_i$ is the set of firm i's customers and $\mathcal{F}_i$ is final demand (i.e., sales outside of the domestic network). Therefore, total sales can be written as

$$\ln S_i = \ln S_i^{net} + \ln \beta_i, \tag{2}$$

where $S_i^{net} \equiv S_i - \mathcal{F}_i$ is network sales and β_i is total sales relative to network sales, $\beta_i \equiv S_i / S_i^{net} \geq 1$, i.e. an inverse measure of (the share of) network sales.

Using equation (1), network sales can be written as

$$\ln S_i^{net} = G + \ln \psi_i + \ln \xi_i, \tag{3}$$

where $\xi_i \equiv \sum_{j \in \mathcal{C}_i} \theta_j \omega_{ij}$. The components ψ_i and ξ_i represent upstream and downstream fundamentals that shape firm size, respectively. To fix ideas, consider the case where sales dispersion is only due to variance in ψ_i. Then, large firms have greater market shares among their customers than small firms (while the number of customers is the same). Next, consider the case where sales dispersion is only due to variance in ξ_i. Then, large firms transact with more, bigger, and/or better-matched customers than small firms (while market shares are the same).

Note that all components of equations (2) and (3) are known: S_i, S_i^{net}, β_i and G come directly from the data, while ψ_i and ξ_i are estimated from equation (1). In order to assess

Table 3: Full vs. Estimation Sample

Full Sample			Estimation Sample			
# Links	# Sellers	# Buyers	Links	Value	Sellers	Buyers
17,304,408	590,271	840,607	99%	95%	74%	88%

Note: Summary statistics for firm-to-firm transactions in the raw B2B data and in the estimation sample.

the role of each margin, we follow the literature (Eaton et al., 2004, Hottman et al., 2016) and perform simple variance decompositions on equations (2) and (3). For the upper-tier decomposition, we regress each component ($\ln S_i^{net}$ and $\ln \beta_i$) separately on log sales. For the lower-tier decomposition, we regress each component ($\ln \psi_i$ and $\ln \xi_i$) separately on log network sales. By the properties of ordinary least squares, and from the exact nature of the decomposition, the coefficients from each decomposition will sum to unity, and the coefficient magnitudes will represent the share of the overall variation in firm size explained by each margin (see Appendix C). All observed and constructed variables are first demeaned by their NACE 4-digit industry average, such that systematic variation across industries is differenced out.

3.2 Results

We first implement the two-way fixed effect regression for firm-to-firm sales from equation (1). The regression is performed on the estimation sample of firm connections from the NBB B2B Transactions Dataset (i.e., not only the core sample of firms that are linked to other datasets, see Section 2.1). As mentioned above, the fixed effects model requires firms to have at least two customers or suppliers to identify the seller or buyer effect, respectively. As the production network is highly connected, we are able to estimate buyer and seller effects for the vast majority of firms in the population. After removing firms with un-identified fixed effects, 99% of the links and 95% of the value of all transactions remain in the estimation sample, see Table 3.

The results from estimating equation (1) are reported in Table 4. Three patterns stand out. First, the adjusted R^2 from the regression is 0.39, indicating that the buyer and seller fixed effects explain a large share of the variation in network activity in the data. Second, the variation in the seller effect $\ln \psi_i$ is larger than that in the buyer effect $\ln \theta_j$. Third, the correlation between the fixed effects is close to zero.

The results in Table 4 inform us about the variation in transaction values, m_{ij}, but not

Table 4: Buyer and Seller Effects

	N	$\frac{var(\ln\psi_i)}{var(\ln\psi_i+\ln\theta_j)}$	$\frac{var(\ln\theta_j)}{var(\ln\psi_i+\ln\theta_j)}$	$\frac{2cov(\ln\psi_i,\ln\theta_j)}{var(\ln\psi_i+\ln\theta_j)}$	Adjusted R^2
$\ln m_{ij}$	17,054,274	0.66	0.32	0.02	0.39

Note: The table reports the (co)variances of the estimated seller and buyer fixed effects from equation (1). The estimation is based on the high-dimensional fixed effects estimator from Correia (2016).

about the variation in firm sales, S_i, which is given by the exact firm sales decomposition in equations (2) and (3). To estimate equations (2) and (3), we use the estimates of $\Psi = \{\psi_i, \theta_j, \omega_{ij}\}$, balance sheet data on total sales S_i, and the final demand ratio β.[11]

Column (1) of Table 5 reports the results from the decomposition of total firm sales from equation (2) on the core sample of firms. Relative differences in final demand across firms, as captured by the ratio of total to network sales, $\ln\beta_i$, account for an economically negligible 1% of the overall variation in firm size. Thus, large firms are not systematically selling relatively more (or less) to final demand than small firms.

Column (2) of Table 5 reports the results from the decomposition of network sales from equation (3). The downstream side accounts for 76 percent of the size dispersion across firms, and upstream fundamentals contribute 24 percent. The upstream factor $\ln\psi_i$ represents the average market share of i among its customers. In other words, being an important supplier to one's customers is only weakly related to overall firm success. These findings suggest that key to understanding the vast firm size heterogeneity observed in modern economies is how firms manage their sales activities, and specifically how they match and transact with buyers in the production network.

3.3 Downstream Decomposition

The initial decomposition results provide evidence for the importance of the downstream margin relative to the upstream and final demand margins in the variance of firm sales. This section further decomposes the downstream margin, and thereby sheds more light on the sources of firm size heterogeneity.

The downstream margin $\xi_i \equiv \sum_{j\in\mathcal{C}_i}\theta_j\omega_{ij}$ can be expressed as

$$\ln\xi_i = \ln n_i^c + \ln\bar{\theta}_i + \ln\Omega_i^c, \tag{4}$$

where n_i^c is the number of customers, and $\bar{\theta}_i \equiv \left(\prod_{j\in\mathcal{C}_i}\theta_j\right)^{1/n_i^c}$ is the average buyer fixed

[11] All firms with estimated fixed effects enter the calculation of $\xi_i \equiv \sum_{j\in\mathcal{C}_i}\theta_j\omega_{ij}$, even if they are not in the balance sheet data themselves.

Table 5: Decomposition.

		(1) Total sales $\ln S_i$		(2) Network sales $\ln S_i^{net}$		(3) Downstream $\ln \xi_i$	
Relative final demand	$\ln \beta_i$	.01	(.00)				
Network sales	$\ln S_i^{net}$	.99	(.00)				
Upstream	$\ln \psi_i$			.24	(.00)		
Downstream	$\ln \xi_i$			.76	(.00)		
# Customers	$\ln n_i^c$					.71	(.00)
Avg Customer Capability	$\ln \bar{\theta}_i$					.03	(.00)
Customer Covariance	$\ln \Omega_i^c$					.26	(.00)

Note: The table reports coefficient estimates from separate OLS regressions of a firm size margin (as indicated in the row heading) on $\ln S_i$, $\ln S_i^{net}$ or $\ln \xi_i$ (as indicated in the column heading). All variables are first demeaned by their 4-digit NACE industry average. The number of firms in the core sample is 94,334. Standard errors in parentheses.

effect among customers. The covariance term Ω_i^c is defined as[12]

$$\Omega_i^c \equiv \frac{1}{n_i^c} \sum_{j \in \mathcal{C}_i} \omega_{ij} \frac{\theta_j}{\bar{\theta}_i}.$$

Each of these components has an intuitive economic interpretation. First, firms face high network demand if they are linked to many customers (high n_i^c). Second, they face high network demand if their average customer has high input purchases (high $\bar{\theta}_i$). Third, they face high network demand if the covariance term Ω_i^c is large, i.e. if large customers (high θ_j) also happen to be good matches (high ω_{ij}). These components are directly available in the data (n_i^c) or can be calculated from $\Psi = \{\psi_i, \theta_j, \omega_{ij}\}$ ($\bar{\theta}_i$ and Ω_i^c).

As with the overall decomposition, we regress each component in equation (4) on $\ln \xi_i$ to evaluate its contribution to the variation in $\ln \xi_i$. The coefficient estimates across components will mechanically sum to one because the left- and right-hand side of equation (4) are by construction identical. As above, all components are demeaned by their 4-digit industry average, so that variation across industries is differenced out.

The last column of Table 5 reports the results for the downstream decomposition. Most of the variation in the downstream component across firms (71 percent) can be attributed to the extensive margin, i.e. the number of (domestic) buyers, $\ln n_i^c$. On the other hand, the average sourcing capability across a firm's customers, $\ln \bar{\theta}_i$, and the customer covariance

[12] By the properties of ordinary least squares, the average term $(1/n_i^c) \sum_{j \in \mathcal{C}_i} \ln \omega_{ij} = 0$ and is therefore omitted from the expression.

term, $\ln \Omega_i^c$, contribute a more modest 3% and 26%, respectively. Therefore, the single most important advantage of large firms is that they successfully match with many buyers, whereas who you match with plays a much smaller role. Appendix C.4-C.5 confirm the robustness of the results and report decomposition results for business groups and by broad economic sectors, respectively.

3.4 Correlations

We conclude this section by documenting the correlations between various firm characteristics in our data. Column 1 in Table 6 shows that firm sales are strongly positively correlated with both the upstream, $\ln \psi_i$, and the downstream, $\ln \xi_i$, components. The correlation coefficient with the downstream component is almost three times as large, mirroring the earlier decomposition. The downstream and upstream components are themselves negatively correlated, implying that firms with many customers or with particularly good customers (leading to a high $\ln \xi_i$) tend to have smaller average market shares among those customers (leading to a low $\ln \psi_i$).[13] Our interpretation of this finding is that firms are unlikely to succeed on both the extensive and the intensive margins: some firms become large by accumulating a customer base, while other firms become large by being important suppliers to their clients, and few firms manage to do both.

These results, coupled with the stylized facts and decomposition findings, are difficult to reconcile with canonical heterogenous-firm models. They suggest that multiple upstream and downstream dimensions of firm activity underpin sales dispersion when firms interact in production networks. One interpretation of our findings is that firm attributes that matter for matching with customers and suppliers are orthogonal, or even negatively related, to firm attributes that determine sales conditional on a match.

4 Theoretical Framework

This section develops the theoretical framework. Our starting point is a model in which firms are heterogeneous in two dimensions. First, firms within an industry have different productivities, which implies that they have different marginal costs and prices.[14] Second, firms have different relationship capabilities. These capabilities determine their ability to match with customers conditional on their (quality-adjusted) prices. We model relationship capability as a relationship fixed cost that the firm must incur for each customer it chooses

[13] Recall from Section 3.3 that the downstream component is large when firms transact with more, bigger and/or better-matched customers.

[14] As is standard in this class of models, productivity and quality are isomorphic.

Table 6: Correlation Matrix

Firm Size Component	$\ln S_i$	$\ln Snet_i$	$\ln \psi_i$	$\ln \xi_i$	$\ln n_i^c$	$\ln n_i^s$
Total Sales, $\ln S_i$	1					
Network Sales, $\ln Snet_i$	.73	1				
Upstream, $\ln \psi_i$	.23	.42	1			
Downstream, $\ln \xi_i$	.66	.83	-.16	1		
# Customers, $\ln n_i^c$	.49	.59	-.33	.85	1	
# Suppliers, $\ln n_i^s$	.76	.57	-.02	.63	.57	1

Note: All correlations are significant at 5%. All variables are demeaned at the NACE 4-digit level.

to serve. A firm with lower relationship fixed costs therefore matches with more customers, all else equal.

Firms operate in a production network and sell to other firms and to final demand. In addition to productivity and relationship capability, a firm's size also depends on its input prices. Input prices are low (and sales high) if the firm has many low-price (or high-quality) suppliers. We first present the model conditional on a set of firm-to-firm links, and subsequently introduce a parsimonious firm-to-firm matching model.

4.1 Technology and Demand

The economy consists of a unit continuum of firms, each with the following production function:

$$y(i) = \kappa z(i)\, l(i)^{\alpha}\, v(i)^{1-\alpha},$$

where $y(i)$ is output (in quantities) of firm i, $z(i)$ is productivity, $l(i)$ is the amount of labor used by firm i, α is the labor share, and $\kappa > 0$ is a normalization constant.[15] $v(i)$ is a constant elasticity of substitution (CES) network input bundle:

$$v(i) = \left(\int_{\mathcal{S}(i)} \nu(k,i)^{(\sigma-1)/\sigma}\, dk \right)^{\sigma/(\sigma-1)},$$

where $\nu(k,i)$ is the quantity purchased from firm k, $\mathcal{S}(i)$ is the set of suppliers to firm i, and $\sigma > 1$ is the elasticity of substitution across suppliers. The corresponding input price index is $P(i) = \left(\int_{\mathcal{S}(i)} p(k)^{1-\sigma}\, dk \right)^{1/(1-\sigma)}$, where $p(k)$ is the price charged by supplier k. The

[15] $\kappa \equiv \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}$. This normalization maps the production function to the cost function, and simplifies the expression for the cost function without any bearing on our results.

wage w is the numéraire. The marginal cost of the firm is then

$$c(i) = \frac{P(i)^{1-\alpha}}{z(i)}. \tag{5}$$

Final Demand. Final consumers also have a CES utility function with the same elasticity of substitution σ. The final consumer is the shareholder of all firms, so that aggregate profits Π become part of consumer income. Income X is therefore the sum of labor income and aggregate profits, $X = wL + \Pi$, where L is inelastically supplied labor.

4.2 Firm-to-Firm Sales

Each firm faces demand from other firms as well as from final demand. Given the assumptions about technology, sales from firm i to j are

$$m(i,j) = p(i)^{1-\sigma} P(j)^{\sigma-1} M(j), \tag{6}$$

where $M(j)$ are total intermediate purchases by firm j, $M(j) = \int_{\mathcal{S}(j)} m(i,j)\,di$.

The market structure is monopolistic competition, so that firms charge a constant markup over marginal costs, $p(i) = \mu c(i)$, where $\mu \equiv \sigma/(\sigma-1)$. After rearranging, sales from i to j can then be written as

$$m(i,j) = \left[\frac{z(i)}{\mu P(i)^{1-\alpha}} P(j)\right]^{\sigma-1} M(j). \tag{7}$$

The model thus delivers a simple log linear expression for firm-to-firm sales, just as in the reduced-form equation (1) in Section 3.

4.3 Equilibrium Conditional on Network

This section characterizes the equilibrium conditional on a fixed network structure. The next section develops the firm-to-firm matching model, and specifies the general equilibrium with endogenous match formation.

To proceed, we introduce additional notation. A firm i is characterized by the tuple $\lambda = (z, F)$, where z is productivity and F is a relationship fixed cost, in units of labor. z and F are potentially correlated, and $dG(\lambda)$ denotes the (multivariate) density of λ. Furthermore, define the link function $l(\lambda, \lambda')$ as the share of seller-buyer pairs (λ, λ') that match.[16]

Backward fixed point. For a given network structure, the equilibrium can be found by solving for two fixed points sequentially. Using the pricing rule $p(\lambda) = \mu c(\lambda)$ and the

[16] Due to idiosyncratic pairwise fixed cost shocks the link function will take values between 0 and 1, see Section 4.4.

equation for marginal costs (5), the input price index can be solved by iterating on a backward fixed point problem:

$$P(\lambda)^{1-\sigma} = \mu^{1-\sigma} \int P(\lambda')^{(1-\sigma)(1-\alpha)} z(\lambda')^{\sigma-1} l(\lambda', \lambda)\, dG(\lambda'). \tag{8}$$

The input costs of firm λ depend on the input costs and productivity of all its suppliers λ', $P(\lambda')$ and $z(\lambda')$.

Forward fixed point. Sales of a type-λ firm are the sum of sales to final and intermediate demand: $S(\lambda) = \mathcal{F}(\lambda) + \int m(\lambda, \lambda') l(\lambda, \lambda') dG(\lambda')$, where $m(\lambda, \lambda')$ now denotes sales by supplier λ to buyer λ'. Final demand is $\mathcal{F}(\lambda) = p(\lambda)^{1-\sigma} \mathcal{P}^{\sigma-1} X$, with the consumer price index equal to $\mathcal{P}^{1-\sigma} = \int p(\lambda)^{1-\sigma} dG(\lambda) = \mu^{1-\sigma} \int P(\lambda)^{(1-\alpha)(1-\sigma)} z(\lambda)^{\sigma-1} dG(\lambda)$. Also note that total input purchases are $M(\lambda) = S(\lambda)(1-\alpha)/\mu$. Using this together with equation (5) yields

$$S(\lambda) = \mu^{1-\sigma} z(\lambda)^{\sigma-1} P(\lambda)^{(1-\sigma)(1-\alpha)} \left(\frac{X}{\mathcal{P}^{1-\sigma}} + \frac{1-\alpha}{\mu} \int \frac{S(\lambda')}{P(\lambda')^{1-\sigma}} l(\lambda, \lambda')\, dG(\lambda') \right). \tag{9}$$

Sales of a type-λ firm depend on final demand, X, the productivity and input price index of the firm itself, $z(\lambda)$ and $P(\lambda)$, as well as the sales and input prices of its customers, $S(\lambda')$ and $P(\lambda')$. Appendix B.1 proves the existence and uniqueness of the equilibrium.

4.4 Firm-to-Firm Matching

We now consider the general equilibrium when the production network is endogenous and sellers match with buyers if and only if the profits from doing so are positive. The seller incurs a relationship fixed cost $F\epsilon$ for every buyer it chooses to sell to, where F varies across sellers, and ϵ is an idiosyncratic component that varies across firm pairs. This matching model is similar to Bernard et al. (2018a) and Lim (2018), but in contrast to these papers, F is a firm-specific attribute that can be correlated with the productivity of the firm, z.[17]

The share of seller-buyer pairs (λ, λ') that match is then

$$l(\lambda, \lambda') = \int I\left[\ln \epsilon < \ln \pi(\lambda, \lambda') - \ln F\right] dH(\epsilon), \tag{10}$$

where $I[]$ is an indicator function, $dH(\epsilon)$ denotes the density of ϵ, and the gross profits from the potential match are

$$\pi(\lambda, \lambda') = \frac{m(\lambda, \lambda')}{\sigma}.$$

[17] In contrast, sales to final demand incur no fixed costs and vary across firms in response to differences in prices (marginal costs).

The introduction of idiosyncratic match costs ϵ is not needed to solve the model or to rationalize the stylized facts presented earlier in the paper. However, ϵ will play a role for the structural estimation in Section 5. Specifically, dispersion in ϵ ensures that the link function is continuous in the parameters of the model, such that standard gradient-based numerical methods can be used to minimize the objective function.

This link function is also a fixed point problem. The gross profits from a potential match, $\pi()$, determine the link probabilities according to equation (10), and the link probabilities determine gross profits via the backward and forward fixed points in equations (8) and (9).

The general equilibrium of the model can be solved by a simple nested fixed point algorithm. (i) Start with a guess of the link function $l()$. (ii) Solve for $P(\lambda)$ and $S(\lambda)$ using the backward and forward fixed points in equations (8) and (9) sequentially. (iii) Calculate gross profits for all potential matches using equation (7). (iv) Calculate the share of seller-buyer pairs (λ, λ') that match according to equation (10). (v) Go back to step (ii) until the link function converges. We do not have a formal proof of existence and uniqueness. In practice, however, the nested fixed point problem is numerically well-behaved and always converges to the same solution irrespective of the chosen starting values.

4.5 Discussion

We conclude the exposition of the model by discussing some key implications and features. We start by considering the role of each dimension of firm heterogeneity in determining equilibrium outcomes on its own. Conditioning on relationship capability, firms with higher productivity have lower marginal costs, lower prices and higher profits from a match with any given buyer, see equation (5). As a result, higher productivity firms match with more buyers, see equation (10), and have greater sales (market share) conditional on a match, see equation (7). Larger total sales and input purchases make higher productivity firms more attractive partners for upstream firms, see equation (7). The increased number of upstream suppliers contributes to an additional reduction in marginal cost through the firm's price index, see equation (8).

Conditioning on productivity, firms with better relationship capability, lower F, are able to match with more buyers, see equation (10), and as a result have greater sales and greater input purchases. As with higher productivity, the greater input demand makes these firms relatively attractive to upstream suppliers, and the greater number of suppliers lowers their marginal cost of production through the input price index. The lower marginal cost results in greater sales (market share) to any given buyer.

Considered by itself, either higher productivity or better relationship capability leads to higher sales through both the extensive margin of more downstream buyers and the intensive margin of greater sales per buyer.

Several features of the model grant it analytical and quantitative tractability, as well as transparency in illustrating the main mechanisms. We consider a unit continuum of firms in the economy. This implies that individual sellers take other sellers' prices and all buyers' input price indices as given when deciding whether to match with a particular buyer and how much to sell to that buyer.

We focus on the costs that sellers incur to match with buyers, and assume that buyers do not face corresponding costs of matching with suppliers. Even with this assumption, in equilibrium the number of both suppliers and buyers varies across firms. This choice gives tractability because firms make separable sales decisions with respect to different buyers and do not internalize the effect of their match decisions on buyers' input demand. It also avoids the well-known problem of interdependence of sourcing decisions in frameworks where buyers choose suppliers, see Antràs et al. (2017).

The model focuses on the domestic production network and does not directly consider the role of exports and imports, both of which are important in the Belgian context. Implicitly exports are included in final demand, even though this almost surely understates the importance of firm-to-firm sales as almost all of export sales are to firms, rather than consumers. Similarly, while we do not model imports, they can be added to production without changing the implications for firm outcomes.[18]

5 Estimation and Results

This section provides a model-based assessment of the origins of firm heterogeneity. Specifically, we exploit the Belgian production network data to parameterize the model above, allowing for heterogeneity in both production and relationship capabilities across firms. We then estimate the model under alternative scenarios to evaluate the quantitative importance of each firm attribute.

5.1 Simulated Method of Moments

The general equilibrium model is estimated by simulated method of moments (SMM). We assume that firm productivity z and relationship capability F are distributed joint log-normal with expectations $\mu_{\ln z} = 0$ and $\mu_{\ln F}$, standard deviations $\sigma_{\ln z}$ and $\sigma_{\ln F}$, and correlation coefficient ρ.[19] The idiosyncratic matching cost ϵ is also assumed log-normal, with mean

[18] See Bernard et al. (2018b) for a static model of a domestic production network with imports in the production function and idiosyncratic match-specific shocks.

[19] The mean of $\ln z$ is not identified, and it is therefore normalized to zero. This normalization is appealing on conceptual grounds. Consider a shift in the productivity distribution, such that productivity increases

$\mu_{\ln \epsilon} = 0$ and variance $var(\ln z + \ln F)$.[20] In sum, there are four unknown parameters to be estimated, $\Upsilon = \{\sigma_{\ln z}, \mu_{\ln F}, \sigma_{\ln F}, \rho\}$.

In addition to the unknown parameters, information is needed on α (labor cost share), μ (markup), and X (aggregate income). α is constructed by dividing total labor costs by total costs in each 4-digit industry and then taking the simple average across industries. μ is constructed by dividing total sales by total costs in each 4-digit industry and then taking the simple average across industries. X is inferred from sales going out of the network, i.e. $X = \sum_i S_i - \sum_i \sum_{j \in \mathcal{C}_i} m_{ij}$. Table 7 summarizes the parameters of the model, their definitions, and the values assigned to them.

We choose four moments in the data to estimate Υ. While all moments jointly pin down all unknown parameters in general equilibrium, there is an intuitive mapping between them. First, the mean log outdegree across firms, $mean(\ln n_i^c)$, helps identify the mean of the relationship costs. Second, the variance of log outdegree, $var(\ln n_i^c)$ and the variance of sales per customer, $var(\ln(S_i^{net}/n_i^c))$, together identify the variances of productivity and relationship costs. Third, the slope coefficient from the regression of average sales per buyer on the number of buyers, $\ln(S_i^{net}/n_i^c) = \alpha + \beta \ln n_i^c + \varepsilon_i$, helps identify the correlation coefficient ρ (see Figure 1): implicitly, a smaller (or more negative) slope coefficient suggests that firms with low relationship costs and therefore high $\ln n_i^c$ are relatively less productive and thus have lower $\ln(S_i^{net}/n_i^c)$.

The parameters to be estimated and the empirical moments are summarized in Table 8. Collecting the empirical moments in the vector x and the corresponding simulated moments in the vector $x^s(\Upsilon)$, the estimates are obtained by solving

$$\arg\min_{\Upsilon} (x - x^s(\Upsilon))' (x - x^s(\Upsilon)).$$

5.2 Results

The estimated parameters are summarized in the first four rows of column (2) in Table 9. The most interesting parameter is the large positive correlation between productivity and relationship costs (ρ). In other words, firms that are more efficient at converting inputs into

for all firms. While this would lower prices and increase welfare, it would not change firms' market shares or the network structure of the economy that are of interest to us.

[20] If dispersion in ϵ is small relative to dispersion in z and F, then the share of links for some (λ, λ') pairs will be close to zero. This complicates the SMM estimation, as the objective function is no longer smooth in the parameters of the model. Choosing $var(\epsilon) = var(\ln z + \ln F)$ ensures that the objective function is sufficiently smooth. This choice does not imply a correlation between ϵ, z or F. This specification is convenient in terms of tractability, but alternative formulations will produce qualitatively similar results as long as the dispersion in ϵ is not too small.

Table 7: Summary of Parameters

Parameter	Definition	Value	Source
α	Labor cost share	.25	Mean of $\frac{\sum_i w_i L_i}{\sum_i (w_i L_i + M_i)}$
μ	Markup	1.22	Mean of $\frac{\sum_i S_i}{\sum_i (w_i L_i + M_i)}$
X	Aggregate final demand	470 bln euros	$\sum_i S_i - \sum_i \sum_{j \in \mathcal{C}_i} m_{ij}$

Table 8: Identification

Moments	Identifies
$mean\,(\ln n_i^c)$	$\mu_{\ln F}$
$var\,(\ln n_i^c)$	$\sigma_{\ln z},\, \sigma_{\ln F}$
$var\,(\ln (S_i^{net}/n_i^c))$	$\sigma_{\ln z},\, \sigma_{\ln F}$
β from regression $\ln (S_i^{net}/n_i^c) = \alpha + \beta \ln n_i^c + \epsilon_i$	ρ

outputs on the production side have lower relationship capability in matching with buyers on the sales side. In addition, the standard deviation of log relationship costs is an order of magnitude larger than the standard deviation of log productivity. To put matching frictions into perspective, we calculate the ratio of relationship costs ($F\epsilon$) to firm-to-firm sales (m) for successful matches in the economy. The mean of this ratio is 0.09, i.e., average relationship costs account for 9 percent of relationship sales. The mean of this ratio across all potential matches is orders of magnitude higher (10,900).

Columns (1) and (2) of Table 9 show moments from the data and the model with the estimated parameters. As expected, the model hits all targeted moments (second panel). In particular, it replicates the negative relationship between the number of customers and the average sales per customer.

The bottom two panels show non-targeted moments on the downstream and upstream sides respectively. The model overpredicts the variance of total sales but is very close on the variance of sales in the network. We also match remarkably well the relationship between the 90th/50th/10th sales percentiles and outdegree documented in Figure 3a, as well as the pattern of negative degree assortativity downstream in Figure 4. Perhaps most importantly, we match the negative sign, if not the magnitude, of the relationship between the firm's average market share in its customers' input purchases and the number of customers in Figure 4b. The data actually suggest a more negative relationship between the two than that found in the model.

The bottom panel of Table 9 reports upstream moments. These moments are interesting

Table 9: SMM Model Fit

	Data	Estimated models		
	(1)	(2) Baseline	(3) noF	(4) noZ
Estimated parameters:				
$\mu_{\ln F}$		16.16	15.08	19.86
$\sigma_{\ln z}$		.19	.19	
$\sigma_{\ln F}$		2.14		1.32
ρ		.95		
Targeted moments:				
$mean\,(\ln n_i^c)$	-8.12	-8.12	-8.18	-8.13
$var\,(\ln n_i^c)$	1.87	1.87	1.95	1.87
$var\left(\ln \frac{S^{net}}{n_i^c}\right)$	2.13	2.13	2.05	.06
β from $\ln \frac{S_i^{net}}{n_i^c} = \alpha + \beta \ln n_i^c + \epsilon_i$	-.23	-.23	1.02	.16
Non-targeted moments:				
<u>Downstream</u>				
$var\,(\ln S_i)$	1.73	2.36	3.34	.69
$var\,(\ln S_i^{net})$	3.12	3.14	8.00	2.53
var (ln value added per worker)	.62	.42	.29	.53
β from $\ln \bar{\delta}_i = \alpha + \beta \ln n_i^c + \epsilon_i$	-.51	-.12	1.27	.21
β from $\ln m_i^k = \alpha + \beta \ln n_i^c + \epsilon_i$	-.25/-.28/-.29	-.22/-.24/-.26	1.07/1.00/1.02	.19/.17/.15
downstream degree assortativity	-.05	-.05	-.10	-.02
<u>Upstream</u>				
$var\,(\ln M_i^{net})$	2.12	2.36	3.34	0.69
$var\,(\ln n_i^s)$	.60	.58	.76	.14
β from $\ln \frac{M^{net}}{n^s} = \alpha + \beta \ln n_i^s + \epsilon_i$	.54	1.02	1.10	1.24
upstream degree assortativity	-.18	-.22	-.16	-.13

Notes: The number of customers and suppliers in column (1), n_i^c and n_i^s, is normalized relative to the number of firms in the final sample. $\bar{\delta}_i$ is the geometric mean of the market share $\delta_{ij} = m_{ij}/M_j$ for seller i across its buyers j. Downstream degree assortativity refers to β from the regression $\ln \text{Mean } n_j^s$ for i's customers$= \alpha + \beta \ln n_i^c + \epsilon_i$. Upstream degree assortativity refers to β from the regression $\ln \text{Mean } n_i^c$ for j's suppliers$= \alpha + \beta \ln n_j^s + \epsilon_j$. $\ln m_i^k$ is the kth (10th/50th/90th) percentile of log sales, $\ln m_{ij}$, for seller i across its customers j. All variables in column (1) except $mean\,(\ln n_i^c)$ are demeaned by NACE 4-digit industry averages.

because they were not targeted in the estimation and because the model itself emphasizes the downstream choices of the seller in finding buyers but leaves aside any choice firms might make about their upstream partners. The model does a good job matching the variance of input purchases from the network, as well as the number of upstream suppliers. Unlike the downstream side, average input purchases per supplier by a buyer are positively related to the number of its suppliers. This feature is replicated by the model, although the model estimate is larger than that in the data. There is also close correspondence between upstream assortativity in the model and in the data: buyers with more suppliers have suppliers who on average have fewer customers in the network.

We also evaluate to what extent the model fits dispersion in labor productivity. In this class of models, value added per production worker, $(S - M)/l$, is constant across all firms within an industry (Hsieh and Klenow, 2009). In our model, however, a firm's employment is the sum of production workers and "marketing" workers, i.e. workers allocated to relationship building (total employment is $L = l + n^c F$). Value added per worker therefore varies across firms and is increasing in value added per marketing worker.[21] In equilibrium, high productivity and high relationship capability firms have higher value added per marketing worker and therefore have greater labor productivity. Table 9 shows that the estimated model produces significant variance in log labor productivity, although dispersion in the data is somewhat higher (.42 versus .62).

For completeness, Tables 10 and 11 report the firm size decomposition, as well as pairwise correlations between the various components implied by the estimated model. According to the estimated model, but in contrast to the data, relative final demand is lower for larger firms. In the model, the volume of sales to final demand is determined by productivity and input costs, but not relationship capability, as there are no fixed costs with matching to consumers (see first part of equation (9)). Given the estimated importance of relationship capability, it is therefore not surprising that the model matches relative final demand poorly. The upstream margin explains more of the variance of network sales in the model (57 percent) compared to the data (24 percent), see Tables 5 and 10, $\ln S_i^{net}$ column. Turning to the correlations, the model matches the negative correlation between the upstream and downstream margins and the negative correlation between the number of customers and the upstream component (see Tables 6 and 11). The model does less well in other dimensions, in particular the positive correlation of the upstream component and sales is too large.

Even though we have added relationship capability to the model, it appears that the role of marginal costs through both productivity and the number of suppliers is still too strong

[21] Value added per worker is $\frac{S-M}{L} = \left(\frac{l+n^c F}{S-M}\right)^{-1} = \left(\frac{l}{S-M} + \frac{n^c F}{S-M}\right)^{-1}$, where the first term is constant across firms and the second term is the inverse of value added per marketing worker.

Table 10: Decomposition: Estimated Model

		Total sales $\ln S_i$		Network sales $\ln S_i^{net}$		Downstream $\ln\xi_i$	
Relative final demand	$\ln\beta_i$	-.08	(.02)				
Network sales	$\ln S_i^{net}$	1.08	(.02)				
Upstream	$\ln\psi_i$			.57	(.03)		
Downstream	$\ln\xi_i$			.43	(.03)		
# Customers	$\ln n_i^c$					1.06	(.00)
Avg Customer Capability	$\ln \bar{\theta}_i$					.04	(.00)
Customer Covariance	$\ln \Omega_i^c$					-.10	(.00)

Note: The table reports coefficient estimates from separate OLS regressions of a firm size margin (as indicated in the row heading) on $\ln S_i$, $\ln S_i^{net}$ or $\ln\xi_i$ (as indicated in the column heading). Standard errors in parentheses.

Table 11: Correlation Matrix: Estimated Model

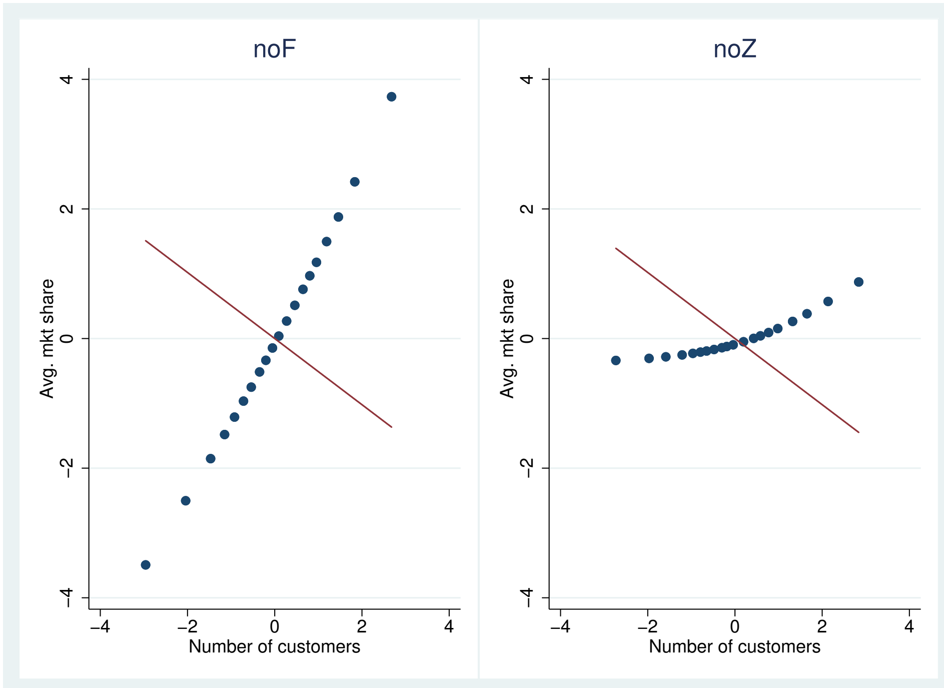
Firm Size Component	$\ln S_i$	$\ln S_i^{net}$	$\ln \psi_i$	$\ln \xi_i$	$\ln n_i^c$	
Total Sales, $\ln S_i$	1					
Network sales, $\ln S_i^{net}$	.94	1				
Upstream, $\ln \psi_i$	.88	.70	1			
Downstream, $\ln \xi_i$	.30	.59	-.16	1		
# Customers, $\ln n_i^c$	.30	.59	-.16	1.00	1	
# Suppliers, $\ln n_i^s$	.99	.94	.88	.30	.30	1

relative to the data. This is clear in the decomposition where the upstream component is too large, in the relationship between market share and the number of customers (more negative in the data), and in the overly large correlations of the upstream component (the number of suppliers) with total sales and network sales.

5.3 A Restricted Model

We next illustrate the need for two firm attributes in order to rationalize observed empirical patterns by estimating a model with heterogeneity in either (i) productivity or (ii) relationship capability but not both. Under (i), there are two parameters to estimate, $\Upsilon = \{\sigma_{\ln z}, \mu_{\ln F}\}$, and we use the same four moments to identify Υ (i.e., the model is over-identified, see Table 8). Under (ii), the parameters to estimate are $\Upsilon = \{\sigma_{\ln F}, \mu_{\ln F}\}$.

Figure 5: Restricted Models



Notes: The left figure shows the binned scatterplot of the number of customers and the average market share in buyers' input purchases in the restricted model where $var\left(F\right)=0$ ("noF"). The right figure shows the binned scatterplot of the number of customers and the average market share in buyers' input purchases in the restricted model where $var\left(z\right)=0$ ("noZ"). The solid line represents the fitted OLS regression line in the data (Figure 4b).

The estimated parameters and fit of these two restricted models is summarized in columns (3) and (4) of Table 9. Both restricted models are unable to generate the negative correlation between average sales per customer and number of customers across sellers (row 4). Figure 5 plots the relationship between sellers' outdegree and average market share of their buyers' input purchases according to the estimated restricted models. In both cases, the model generates the opposite pattern to the empirical regularity in Figure 3. In addition, the single factor models do a relatively poor job of matching the variances of total sales and network sales.

6 Conclusions

This paper quantifies the origins of firm size heterogeneity when firms are interconnected in a production network. We report three stylized facts from the production network data that motivate the subsequent analysis and model. First, the enormous dispersion found in sales across firms is also found in the production network in terms of firm-to-firm connections and the value of pairwise sales. Second, firms with more customers have higher sales but lower average sales per customer and lower market shares (of input purchases) among their customers. Finally, there is negative degree assortativity between buyers and suppliers, i.e. sellers with more customers match with customers who have fewer suppliers on average.

Taken together, these facts present challenges to many existing models of firm heterogeneity. The large variation in sales across firms within an industry is intuitively related to variation in the number of customers: larger firms have more customers. However, larger firms also sell less to their customers. Models that emphasize heterogeneity in productivity across firms cannot explain these facts simultaneously. In particular, such models imply that firms with more customers should also sell more to each of their customers and have higher rather than lower market shares.

We confirm the importance of the production network in a decomposition of the variance of firm sales within narrowly defined industries. 76 percent of the variation in firm sales is associated with the downstream component, and most of that is due to variation in the number of customers. The upstream component contributes around one fourth of the total variation, and variation in the share of sales outside the domestic production network plays a minor role. These findings imply that trade in intermediate goods and the number of firm-to-firm connections are essential to understanding firm performance and, consequently, aggregate outcomes.

Motivated by the stylized facts and decomposition results, we develop a quantitative general equilibrium model of firm-to-firm trade. In the model, firms differ along two dimensions – productivity and relationship capability – defined respectively as production efficiency and (the inverse of) the fixed cost of matching with a customer. Suppliers match with customers if the gross profits of the match exceed the supplier-specific fixed matching cost. Marginal costs, employment, prices, and sales are endogenous outcomes because they depend on the outcomes of all other firms in the economy. A link between two firms increases the total sales of both the seller and the buyer; for the seller this occurs mechanically because it gains a customer, while for the buyer this arises because a larger supplier base lowers the marginal cost of production.

We estimate parameters of the model using simulated method of moments. The results reveal a strong negative correlation between the two firm characteristics. Firms with higher productivity have lower relationship capability. Importantly, both dimensions of firm heterogeneity are necessary to match the data: shutting down one at a time results in poor model fit, including the inability to replicate the negative relationship between the number of customers and average sales per customer.

Our results challenge current understanding of the sources of firm size heterogeneity, and point to important areas for future research on the negative relationship between firm productivity and relationship capability. While we make progress in matching the relative importance of upstream and downstream factors in firm success, there is room for new models to better fit these features of the production network. In addition, research is needed to examine the within firm factors that lead to a negative relationship between productivity and relationship capability. One promising avenue for further work is examining span of control issues inside the firm and the allocation of resources to improving productivity versus acquiring more customers.

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Appendix

A Data Sources and Data Construction

A.1 Data sources

The empirical analysis draws on three main data sources administered by the National Bank of Belgium (NBB): (i) the NBB B2B Transactions Dataset, (ii) annual accounts from the Central Balance Sheet Office at the NBB supplemented by VAT declarations, and (iii) the Crossroads Bank at the NBB. Firms are identified by a unique enterprise number, which is common across all databases and allows for unambiguous merging.

Firm-to-firm relationships The confidential NBB B2B Transactions Dataset contains the value of yearly sales relationships among all VAT-liable Belgian enterprises for the years 2002 to 2014, and is based on the VAT listings collected by the tax authorities. All firms that deliver goods or services as an economic activity, on a regular and independent basis, are VAT liable. These enterprises have to charge an ad valorem tax on their sales and can recover VAT paid on their purchases. This includes foreign companies with a branch in Belgium and firms whose securities are officially listed in Belgium. Enterprises that only perform financial transactions, medical or socio-cultural activities such as education are exempt. Firms with sales less than 15,000 euro can choose to be exempt from the VAT liabilities. The standard VAT rate in Belgium is 21%, but for some goods a reduced rate of 12% or 6% applies.x[22] At the end of every calendar year, all VAT-liable enterprises have to file a complete listing of their Belgian VAT-liable customers over that year.[23] An observation in this dataset refers to the value of sales in euros by enterprise i to enterprise j within Belgium, excluding the VAT due on these sales. The reported value is the sum of invoices from i to j in a given calendar year. Whenever this aggregated value is 250 euros or greater, the relationship has to be reported.[24] Fines for late or incomplete reporting ensure a very high quality of the data. Note that each relationship is directed, as the observation from i to j is different from the observation from j to i; i.e. firm i might be both a supplier to and a customer of j. The dataset thus covers both the extensive and the intensive margins of the Belgian production network. A detailed description of the collection and cleaning of this dataset is given in Dhyne et al. (2015).

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22 See ec.europa.eu/taxation_customs for a complete list of rates. These rates did not change over our sample period.

23 Sample VAT listings forms can be found at here (French) and here (Dutch).

24 Pecuniary sanctions are given to firms for late or erroneous reporting.

Firm-level characteristics We extract information on enterprises' annual accounts from the Central Balance Sheet Office at the NBB for the years 2002 to 2014. Enterprises above a certain size threshold have to file annual accounts at the end of their fiscal year.[25] We retain information on the enterprise identifier (VAT ID), turnover (total sales in euros, code 70 in the annual accounts), input purchases (total material and services inputs in euros and net changes in input stocks, codes 60+61), labor cost (total cost of wages, social securities and pensions in euros, code 62), and employment (average number of full-time equivalent (FTE) employees, code 9087). We annualize all flow variables from fiscal years to calendar years by pro-rating the variables on a monthly basis. [26][27]

Enterprises below a size threshold can report abbreviated annual accounts. These firms report labor cost and employment, but are not required to report turnover or input purchases. For these small enterprises, we supplement information on turnover and inputs from their VAT declarations. All VAT-liable enterprises have to file periodic VAT declarations with the tax administration.[28] The VAT declaration contains the total sales value (including domestic sales and exports), the VAT amount charged on those sales (both to other enterprises and to final consumers), the total amount paid for inputs sourced (including domestic and imported inputs), and the VAT paid on those input purchases. This declaration is due monthly or quarterly depending on firm size, and it is the basis for the VAT due to the tax authorities every period. We aggregate the VAT declarations to the annual frequency.

We obtain information on the main economic activity of each enterprise at the NACE 4-digit level from the Crossroads Bank of Belgium for the years 2002 to 2014. We concord NACE codes over time to the NACE Rev. 2 version to deal with changes in the NACE classification over our panel from Rev. 1.1 to Rev. 2. Table 12 lists industry groups at the NACE 2-digit level.

A.2 Data construction

We calculate the final demand for enterprise i in year t as i's total salesminus the value of all of its B2B sales. The B2B Transactions dataset contains all seller-buyer relationships, including both intermediate and investment goods. This implies that final demand contains final domestic consumption, exports, and tiny business transactions below 250 euros that

[25] See here for filing requirements and exceptions. See here for the size criteria and filing requirements for either full-format or abridged annual accounts.

[26] In our data, 78% of firms have annual accounts that coincide with calendar years, while 98% of firms have fiscal years of 12 months.

[27] Total input purchases are the sum of material and service inputs, and include both new inputs and net changes in input stocks. Employment is reported as average full-time equivalent employees. Total labor costs include wages, social security, and pension contributions.

[28] Sample VAT declaration forms can be found at here (French) and here (Dutch).

Table 12: NACE classification of industry groups (NACE Rev 2)

NACE Section	NACE Division	Description	Industry
A	NACE 01-03	Agriculture, forestry and fishing	Primary and Extraction
B	NACE 05-09	Mining and quarrying	Primary and Extraction
C	NACE 10-33	Manufacturing	Manufacturing
D	NACE 35	Electricity, gas, steam and air conditioning supply	Utilities
E	NACE 36-39	Water supply; sewage, waste management and remediation activities	Utilities
F	NACE 41-43	Construction	Construction
G	NACE 45-47	Wholesale and retail trade; repair of motor vehicles and motorcycles	Market Services
H	NACE 49-53	Transportation and storage	Market Services
I	NACE 55-56	Accommodation and food service activities	Market Services
J	NACE 58-63	Information and communication	Market Services
K	NACE 64-66	Financial and insurance activities	Market Services
L	NACE 68	Real estate activities	Market Services
M	NACE 69-75	Professional, scientific and technical activities	Market Services
N	NACE 77-82	Administrative and support service activities	Market Services
O	NACE 84	Public administration and defence; compulsory social security	Non-Market Services
P	NACE 85	Education	Non-Market Services
Q	NACE 86-88	Human health and social work activities	Non-Market Services
R	NACE 90-93	Arts, entertainment and recreation	Non-Market Services
S	NACE 94-96	Other service activities	Non-Market Services
T	NACE 97-98	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use	–
U	NACE 99	Activities of extraterritorial organizations and bodies	–

are not observed in the B2B dataset. Similarly, we calculate i's total input purchases from the network as the value of all of its B2B purchases. We infer i's input purchases from outside the network as its total input purchases minus its total B2B network purchases. This residual, unobserved part of input expenditures, contains imports and unobserved B2B input purchases under 250 euro. We drop firms that have missing employment information or less than one FTE employee, as these may be sole traders, shell companies or management companies.

Labor shares are calculated at the 4-digit NACE level, as the sum of labor cost over labor cost plus total input usage across all firms within that sector: $\alpha_S = \frac{\sum_j wL_j}{\sum_j (wL_j + M_j)}$, where wL_j is firm j's wage bill and M_j its total expenditure on intermediate inputs (both from the annual accounts) for all j in sector S.

B Theory Appendix

B.1 Existence and Uniqueness

We prove existence and uniqueness by showing that the fixed network equilibrium belongs to the class of models analyzed by Allen et al. (2016).

Allen et al. (2016) consider the following system of equations:

$$\prod_{h=1}^{K} \left(x_i^h\right)^{\gamma_{kh}} = c_i^k + \sum_{j=1}^{N} K_{ij}^k \prod_{h=1}^{K} \left(x_j^h\right)^{\beta_{kh}},$$

where $i, j \in \{1, .., N\}$ are firms/sectors, x_i^h is the type h equilibrium variable, c_i^k is a constant and K_{ij}^k are exogenous linkages between i and j. With $K = 1$ this reduces to

$$x_i^\gamma = c_i + \sum_{j=1}^{N} K_{ij} x_j^\beta. \tag{11}$$

The backward fixed point in equation (8) can be written in the form of equation (11) with $\gamma = 1$, $c_i = 0$, $\beta = 1 - \alpha$ and $K_{ij} = z\left(\lambda'\right)^{\sigma-1}$. Using their notation, $\mathbf{A}$ is simply $1 - \alpha$ and therefore the maximum eigenvalue of $\mathbf{A}^p$ is also $1 - \alpha < 1$. According to their Theorem 2(i), there exists a unique and strictly positive solution to the backward fixed point.

The forward fixed point in equation (9) can be written in the form of equation (11) with $\gamma = 1$, $c_i = \mu^{1-\sigma} z\left(\lambda\right)^{\sigma-1} P\left(\lambda\right)^{(1-\sigma)(1-\alpha)} X/\mathcal{P}^{1-\sigma}$, $\beta = 1$ and $K_{ij} = \left(1-\alpha\right)/\left(\mu P\left(\lambda'\right)^{1-\sigma}\right)$. Using their notation, $\mathbf{A}$ is 1 and therefore the maximum eigenvalue of $\mathbf{A}^p$ is also 1. According to their Theorem 2(ii.a) there exists at most one strictly positive solution to the forward fixed point.

C Additional Results and Robustness

This section first discusses the empirical relevance of the functional form chosen for firm-to-firm sales in equation (1). It then examines the necessary assumptions on the assignment process of buyers and sellers for OLS to identify the underlying parameters of interest, and develops a test for conditional exogenous mobility in the context of a production network. Finally, it presents more evidence on the robustness of the results.

C.1 Functional form

The log-linear relationship in equation (1) predicts the following: (i) expected sales from seller i to customer j are increasing in the average sales of i to other customers k; (ii) expected purchases by buyer j from seller i are increasing in the average purchases by j from other suppliers k.

Properties (i)-(ii) can be tested non-parametrically as follows. For each seller i and buyer j, calculate the leave-out mean of log sales ($\bar{s}_i^{-l}$) and purchases ($\bar{m}_j^{-l}$) across its buyers and suppliers, excluding customer/supplier l, respectively:[29]

$$\bar{s}_i^{-l} = \frac{\sum_{j \in \mathcal{C}_i \setminus l} \ln m_{ij}}{n_i^c - 1}$$

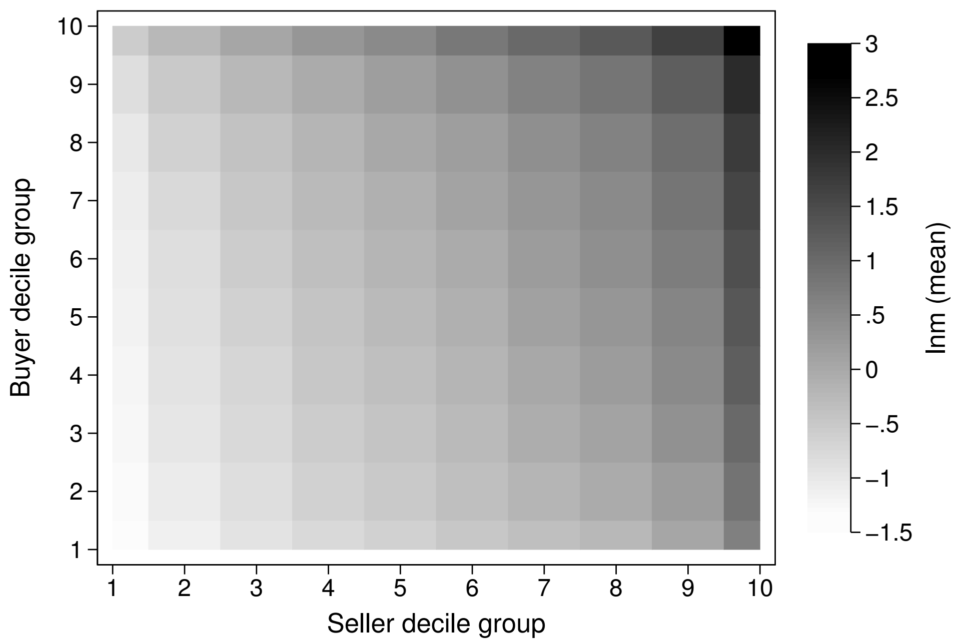
$$\bar{m}_j^{-l} = \frac{\sum_{k \in \mathcal{S}_j \setminus l} \ln m_{kj}}{n_j^s - 1}.$$

Then sort firms into decile groups based on $\bar{s}_i^{-l}$ and $\bar{m}_j^{-l}$, denoting the decile group the firm belongs to as $q_{\bar{s}} = 1, .., 10$ and $q_{\bar{m}} = 1, .., 10$, respectively. Finally, calculate the mean of $\ln m_{ij}$ for every decile group pair, $\overline{\ln m}_{q_{\bar{s}}, q_{\bar{m}}}$, e.g. the average $\ln m_{ij}$ for the seller-buyer pairs in $(q_{\bar{s}}, q_{\bar{m}}) = (1, 1)$, and so on.

Figure 6 illustrates the results using a heatmap. The decile groups $q_{\bar{s}}$ and $q_{\bar{m}}$ are plotted on the horizontal and vertical axes, respectively. $\ln m_{ij}$ is increasing in the average sales from i to other customers k (moving from left to right in the diagram), and $\ln m_{ij}$ is increasing in the average purchases of j from other suppliers k (moving from bottom to top in the diagram).

[29] Using the overall mean generates a mechanical relationship between e.g. seller size and sales between i and j. We calculate $\bar{s}_i^{-l}$ and $\bar{m}_j^{-l}$ for all (i, l) and (j, l) pairs respectively. Firms with only one customer or supplier are by construction omitted from the sample.

Figure 6: Average log sales across seller and buyer decile groups



Note: The figure shows the average of $\ln m_{ij}$ in all decile group pairs $(q_{\bar{s}}, q_{\bar{m}})$.

C.2 Assumptions on the Assignment Process

Equation (1) is a two-way fixed effects model similar to the models that are used in the employer-employee literature (Abowd et al., 1999; Card et al., 2013).[30] OLS estimates of $\ln \psi_i$ and $\ln \theta_j$ will identify the effect of seller and buyer characteristics if the following moment conditions are satisfied:

$$\begin{cases} E[s_i' r] = 0 & \forall i \\ E[b_j' r] = 0 & \forall j. \end{cases} \tag{12}$$

Here $S = [s_1, ..., s_N]$ is the $N^* \times N_s$ seller fixed effects design matrix, $B = [b_1, ..., b_N]$ is the $N^* \times N_b$ buyer fixed effects design matrix, r is the $N^* \times 1$ vector of residual match effects, and N^*, N_s and N_b are the number of matches, sellers and buyers, respectively. The first condition states that for each seller i, the average $\ln \omega_{ij}$ across buyers j is zero, while the second condition states that for each buyer j, the average $\ln \omega_{ij}$ across sellers i is zero. Intuitively, a high $\ln \omega_{ij}$ that is common across customers j of i will be automatically loaded onto i's seller effect (and similarly for suppliers i of j). In other words, these moment conditions require that the assignment of suppliers to customers is exogenous with respect to ω_{ij}, so-called conditional exogenous mobility in the labor literature.

It is instructive to review four cases when these moment conditions hold. First, they hold if firms match based on their seller and buyer effects, e.g., highly productive firms match with more and/or different customers/suppliers than less productive ones. Second, the assumption holds if firms match based on idiosyncratic pair-wise shocks that are unrelated to $\ln \omega_{ij}$. One example of this is idiosyncratic fixed costs, such as costs related to search and matching, which affect profits for a potential match but not the value of bilateral sales.[31]

Now consider the case of endogenous mobility. To fix ideas, assume that matching is based on the idiosyncratic match component of sales, ω_{ij}, together with the seller effect ψ_i. In that case, only high ψ_i sellers would want to match with low ω_{ij} buyers. OLS would then give a downward bias in the estimated ψ_i, because OLS imposes that the average $\ln \omega_{ij}$ across customers is zero.

To explore the possibility that matching shocks are correlated with sales shocks, we test conditional exogenous mobility as follows. Consider firm i selling to customers 1 and 2. The

[30] The linear fixed-effects approach imposes no restrictions on the seller and buyer effects, unlike random or mixed effects models. With random effects, one also needs to model the network formation game to assess the plausibility of the required distributional assumptions for unobserved heterogeneity (see Bonhomme (Forthcoming)).

[31] E.g., the matching framework presented in Section 4.

expected difference in bilateral sales is

$$\Delta \ln m_i \equiv E\left[\ln m_{i2} - \ln m_{i1} \mid (i,1),(i,2)\right] = \ln \theta_2 - \ln \theta_1 + E\left[\ln \omega_{i2} - \ln \omega_{i1} \mid (i,1),(i,2)\right].$$

Consider the case $\theta_2 > \theta_1$. Under exogenous mobility, the last expectation term is zero, and $\Delta \ln m_i$ is unrelated to firm i characteristics. Under endogenous mobility, the last expectation term is non-zero, and $\Delta \ln m_i$ is potentially a function of firm i characteristics. Now seller i will only want to match with customer 1 if ω_{i1} is sufficiently large. The expectation $E\left[\ln \omega_{i2} - \ln \omega_{i1} \mid (i,1),(i,2)\right]$ is then negative. Moreover, for small sellers (low ψ_i), the size of ω_{i1} is important for whether a match occurs or not, while for large sellers (high ψ_i), the size of ω_{i1} is less important (since matching is determined by both ψ_i and ω_{ij}). Under endogenous mobility, the expectation is therefore less negative for high-ψ_i than low-ψ_i firms, so that $\Delta \ln m_i$ is greater for high-ψ_i than low-ψ_i firms. Under exogenous mobility, by contrast, $\Delta \ln m_j$ should be unrelated to ψ_i.[32]

Going back to the seller and buyer decile groups constructed above, these predictions can be tested by looking at $\overline{\ln m}_{q_{\bar{s}},q_{\bar{m}}}$ when moving from a small to a big customer, for different groups of sellers. Figure 7 shows the results. Each line represents the mean of log sales for a given seller decile group (1,..,10). Within a seller group, we calculate $\overline{\ln m}_{q_{\bar{s}},q_{\bar{m}}}$ to small customers (buyer decile group 1) and to big customers (buyer decile group 10). Under exogenous mobility, those lines should be parallel, i.e. for buyer bins $q_{\bar{m}}$ and $q'_{\bar{m}}$, $\overline{\ln m}_{q_{\bar{s}},q_{\bar{m}}} - \overline{\ln m}_{q_{\bar{s}},q'_{\bar{m}}}$ does not depend on the seller decile group. The lines are, to a large degree, parallel, in particular for the seller decile groups 2 to 9. Parallel lines are a sufficient but not necessary condition for exogenous mobility: If the data generating process is not linear in logs, then one could find non-parallel lines even under exogenous mobility.

One can test for this non-parametrically as follows. Using the buyer and seller bins defined above, exogenous mobility implies that

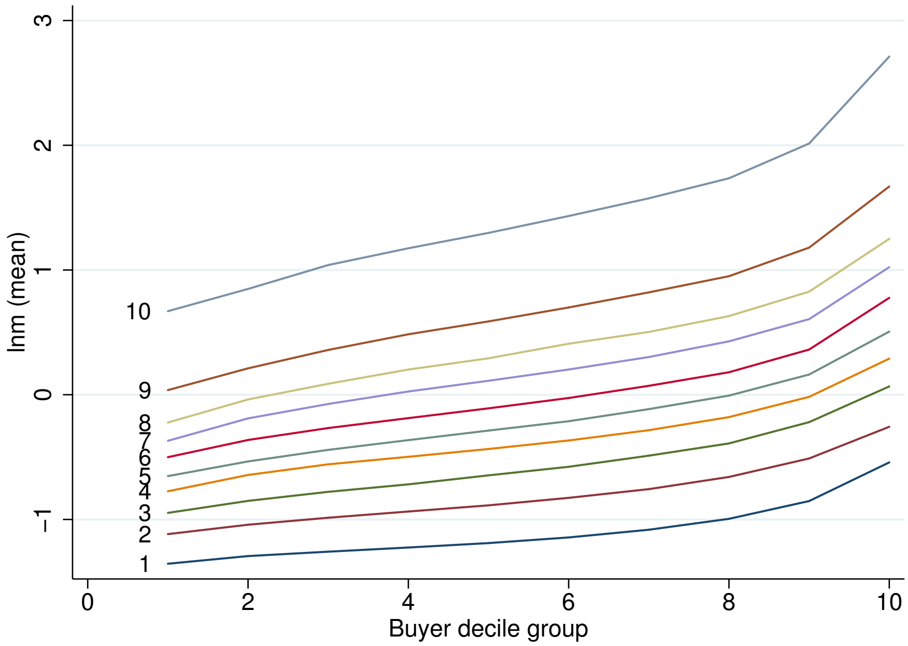
$$\overline{\ln m}_{q'_{\bar{s}},q'_{\bar{m}}} - \overline{\ln m}_{q'_{\bar{s}},q_{\bar{m}}} - \left(\overline{\ln m}_{q_{\bar{s}},q'_{\bar{m}}} - \overline{\ln m}_{q_{\bar{s}},q_{\bar{m}}}\right) = 0, \tag{13}$$

for any bins $q_{\bar{s}}$, $q'_{\bar{s}}$, $q_{\bar{m}}$ and $q'_{\bar{m}}$. We form these averages for $q'_{\bar{s}} = q_{\bar{s}} + 1$ and $q'_{\bar{m}} = q_{\bar{m}} + 1$ and test the null hypothesis that the double difference equals zero. This yields 81 separate hypothesis tests across all buyer-seller pair bins.[33] Overall, the results mirror those in Figure 7: the double differences are not significantly different from zero in the middle of the distribution, whereas we find significant deviations in the tails. Significant deviations are typically relatively small: e.g. moving from a 6th to 7th decile buyer yields 12% more sales

32 Card et al. (2013) test for endogenous mobility for employer-employee matches using a related, but different, test.

33 t-values are calculated using Welch's t-test.

Figure 7: Average log sales across seller and buyer decile groups



Note: The figure shows $\ln \bar{m}_{ij}$ across buyer decile groups $q_{\bar{m}} = 1, .., 10$. Each line represents a seller decile group, $q_{\bar{s}} = 1, ., 10$.

for seller decile 9 and 14% more sales for seller decile 10. All 81 hypothesis tests are reported in Appendix C.3.

C.3 Exogenous mobility test

We report 81 separate hypothesis tests across all buyer-seller pair bins in Table 13. Each column refers to the change from buyer decile t to $t+1$, and each row refers to the change from seller decile t to $t+1$. For example, the cell (3-2,2-1) reports the difference $\overline{\ln m}_{3,2} - \overline{\ln m}_{3,1} - \left(\overline{\ln m}_{2,2} - \overline{\ln m}_{2,1}\right)$.

Table 13: Exogenous mobility test

		Buyer decile								
		2-1	3-2	4-3	5-4	6-5	7-6	8-7	9-8	10-9
Seller decile	2-1	0.01*	0.02*	0.02*	0.03*	0.02*	0.01	0.01*	0.01	-0.04*
	3-2	0.04*	0.01*	0.01	0.00	0.00	0.01*	0.00	0.01	0.02*
	4-3	0.01	0.01*	0.02*	0.00	0.01	0.00	0.01	0.00	0.03*
	5-4	0.00	-0.01	0.00	0.01*	0.00	0.00	0.00	0.00	0.08*
	6-5	0.02*	0.02*	0.01	0.00	0.01	0.01	0.02*	0.00	0.02*
	7-6	0.04*	0.02*	0.02*	0.01*	0.01*	0.01	0.01	0.03*	0.00
	8-7	0.00	0.01*	0.00	0.00	0.03*	-0.01	0.00	-0.01	0.01
	9-8	-0.01	0.02*	0.02*	0.01	-0.01	0.02*	0.01	0.05*	0.08*
	10-9	0.03*	0.04*	0.01	0.02*	0.02*	0.02*	0.02*	0.04*	0.19*

Note: The table shows the double difference from equation (13) in the main text. Significance: * $<$ 5%, ** $<$ 1%, *** $<$0.1%. t values are based on Welsh's t-test.

C.4 Business Groups

There remains the possibility that intra-firm trading and ownership structure across VAT enterprises is affecting the results. In particular, while the VAT ID is the legal entity of a firm in Belgium, some firms might be owned by other firms, generating intra-firm trade between parents and affiliates, which might not be subject to typical market forces. While the VAT ID is typically used in firm-level analysis of Belgian data (see Amiti et al., 2014, Magerman et al., 2016 and Bernard et al., 2019b), we follow the procedure in Tintelnot et al. (2017) and aggregate variables across multiple VAT IDs owned by the same firm as a robustness check. VAT IDs are grouped into a single firm if the same parent company owns at least 50% of their shares. Turnover, inputs, employment and labor costs are summed across subsidiaries to the group level, after subtracting within group transactions from turnover and inputs to avoid double counting. The NACE code of the firm with the largest turnover is assigned to the group. There are 11,737 groups with multiple VAT IDs in the raw data in 2014, but they account for a sizable fraction of output. The resulting decomposition is almost identical across all components, e.g. the overall decomposition in Table 14.

C.5 Results by sector

We have also explored the stability of our results across different sectors by performing the decomposition exercise separately for six broad sector groups. Across the board, the estimated coefficients are relatively close to the baseline findings in the main paper. One

Table 14: Business Groups.

		Total sales $\ln S_i$		Network sales $\ln S_i^{net}$		Downstream $\ln \xi_i$	
Relative final demand	$\ln\beta_i$	.01	(.00)				
Network sales	$\ln S_i^{net}$	.99	(.00)				
Upstream	$\ln\psi_i$			.22	(.00)		
Downstream	$\ln\xi_i$			.78	(.00)		
# Customers	$\ln n_i^c$					.72	(.00)
Avg Customer Capability	$\ln \bar{\theta}_i$					.03	(.00)
Customer Covariance	$\ln \Omega_i^c$					.26	(.00)

Note: The table reports coefficient estimates from OLS regressions of a firm size margin (as indicated in the column heading) on total firm sales. All variables are first demeaned by their 4-digit NACE industry average. Standard errors in parentheses.

exception is construction (NACE 41 to 43), where the final demand term β_i enters with a coefficient of -0.10. However, this is expected, as large construction firms typically sell relatively less to final demand compared to small construction firms. A further decomposition by 2-digit and by 4-digit industries confirm the ranking of the components in contribution to sales variance (results available on request).

C.6 Variance decompositions

This section derives statistical properties of the baseline variance decomposition. Consider the following identity:

$$s \equiv \sum_k a_k.$$

The variance of s is

$$var\,(s) = \sum_k \sigma_{kk} + \sum_k \sum_{i \neq k} \sigma_{ki}, \tag{14}$$

where $\sigma_{ki} = cov\,(a_k, a_i)$. In the baseline decomposition, we regress each element a_k on s. By the properties of OLS, the estimate is

$$\beta_k = \frac{cov\,(a_k, s)}{var\,(s)} = \frac{1}{var\,(s)} \left(\sigma_{kk} + \sum_{i \neq k} \sigma_{ki} \right). \tag{15}$$

Note that the sum of all β_k's equals one,

$$\sum_k \beta_k = \frac{1}{var\,(s)} \left(\sum_k \sigma_{kk} + \sum_k \sum_{i \neq k} \sigma_{ki} \right) = 1.$$

Table 15: Variance decomposition by broad sectors.

Industry	N	$\ln Snet_i$	$\ln\beta_i$	$\ln\psi_i$	$\ln\xi_i$	$\ln n_i^c$	$\ln\bar{\theta}_i$	$\ln\Omega_i^c$
Primary & Extraction	2,838	1.01***	−.01	.28***	.72***	.73***	−.03***	.30***
Manufacturing	16,905	.96***	.04***	.25***	.75***	.71***	.03***	.26***
Utilities	852	.95***	.05**	.17***	.83***	.70***	−.01*	.32***
Construction	19,008	1.10***	−.10***	.21***	.79***	.66***	.06***	.28***
Market Services	53,582	.96***	.04***	.24***	.76***	.73***	.02***	.25***
Non-Market Services	1,149	1.02***	-.02	.27***	.73***	.72***	.04***	.24***

Note: The table reports coefficient estimates from OLS regressions of a firm size margin (as indicated in the column heading) on total firm sales. Significance: * < 5%, ** < 1%, *** <0.1%.

Also note that in the case with only two components, the covariance term in equation (15) is split equally among components:

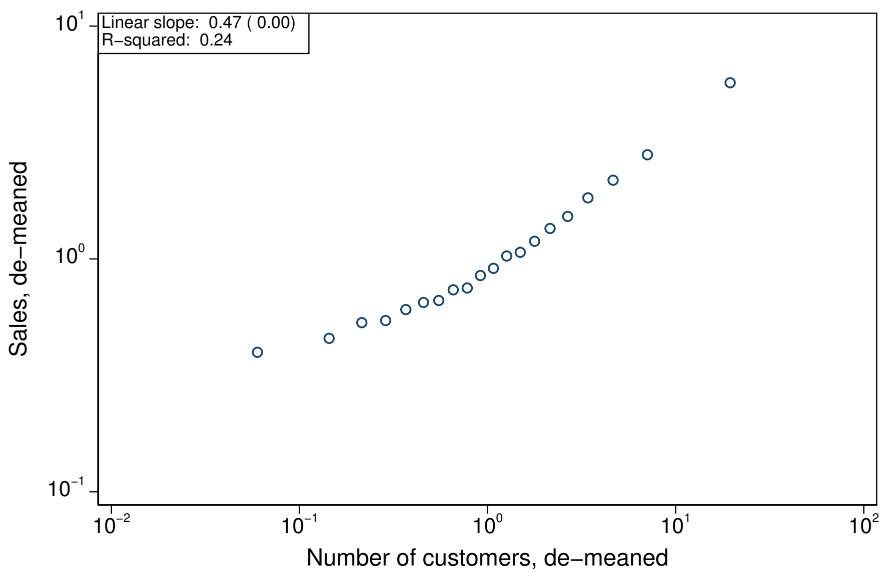
$$\beta_1 = (\sigma_{11} + \sigma_{12}) / var(s)$$
$$\beta_2 = (\sigma_{22} + \sigma_{12}) / var(s) .$$

C.7 Additional correlations

First, we document the relationship between total sales and the number of customers in Figure 8. As with network sales, total sales increases with the number of customers, with an implied slope of 0.47.

Second, we document the relationship between the number of suppliers and network purchases in Figures 9 and Figure **??**. These graphs mirror the figures on sales and number of customers in the main text: total input purchases as a function of the number of suppliers to j, input expenditure per supplier as a function of the number of suppliers, and average upstream market share as a function of the number of suppliers. Average upstream market share refers to the geometric mean of the market share m_{ij}/S_i of firm j in the total sales of its suppliers i.

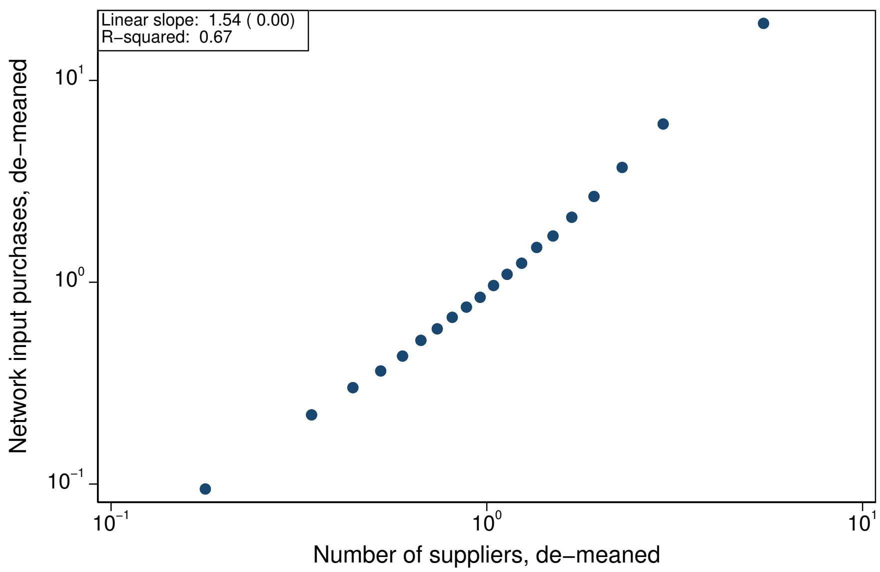
Figure 8: Total sales and Number of Customers



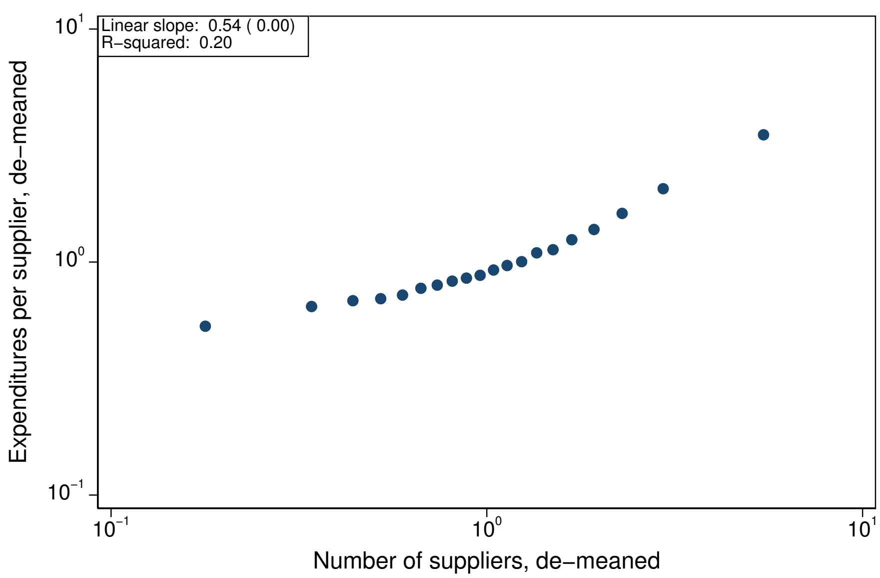
Note: The binned scatterplots groups firms into 20 equal-sized bins by the log number of customers and computes the mean of the variables on the x- and y-axes. Firm sales refer to sales to both customers inside and outside the domestic production network. All variables are demeaned by NACE 4-digit industry averages. Implied elasticities and R-squared from OLS regressions with NACE 4-digit industry fixed effects are reported in the corner of the graph.

Figure 9: Total Network Purchases, Average Purchases and Number of Suppliers

(a) Total Input Purchases.



(b) Average Sales



Note: The binned scatterplots group firms into 20 equal-sized bins by log number of suppliers, and compute the mean of the variables on the x- and y-axes in each bin. All variables are demeaned by NACE 4-digit industry averages. Implied elasticities and R-squared from OLS regressions with NACE 4-digit industry fixed effects are reported in the corner of each graph.